

# Formal Development of Cyber-Physical Systems: The Event-B Approach

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# About Me

- Junior Research Assistant (2014-16) on the SafeCap project: formal methods for a safe and optimum railway,
- PhD work (2016-20, iCase w. Siemens Rail Automation) on formal engineering of heterogeneous railway signalling systems,
- Post-doctoral work (2020-) on the integration of hybridised Event-B and reachability analysis, real-time reachability analysis of autonomous systems and safe AI.

# Cyber-Physical Systems

What are Cyber-Physical Systems (CPS)?

- integrate **computation** and **physical** processes,
- **networked** computers control physical systems.

Examples of CPS can be found in many industry sectors<sup>1, 2</sup>:



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<sup>1</sup> <https://www.phillymag.com/healthcare-news/2019/07/15/medcrypt-hack-proof-medical-devices/>

<sup>2</sup> <https://sites.rmit.edu.au/cyber-physical-systems/>

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Examples of CPS can be found in many industry sectors.

Importantly many of these systems are **safety-critical**.

# Cyber-Physical Railway Signalling Systems

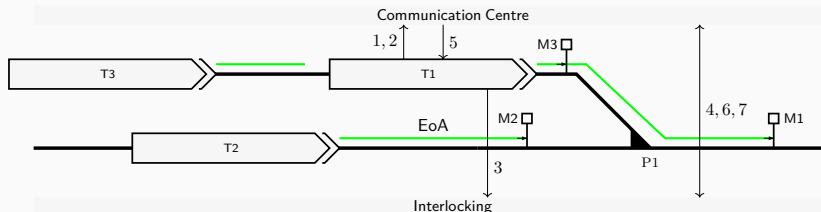


<sup>2</sup> Photo taken from [https://www.networkrail-training.co.uk/media/Signaller\\_Training.01.jpeg](https://www.networkrail-training.co.uk/media/Signaller_Training.01.jpeg)

# Cyber-Physical Railway Signalling Systems

Railway signalling systems are safety-critical cyber-physical systems:

- European Train Control System (ETCS L0-3, part of ERTMS),  
Communication-based Train Control (CBTC),

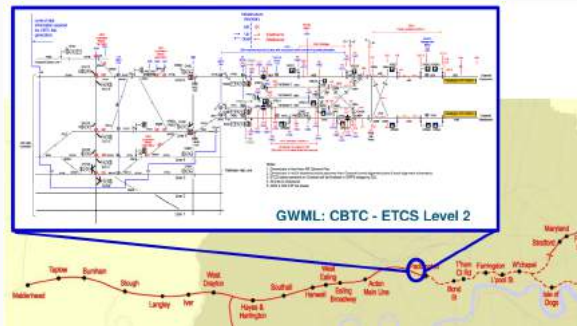


Trains are **hybrid systems** (discrete and continuous behaviour)

# Cyber-Physical Railway Signalling Systems

Railway signalling systems are safety-critical cyber-physical systems:

- European Train Control System (ETCS L0-3, part of ERTMS),  
Communication-based Train Control (CBTC),
- **Heterogeneous** railway signalling networks (Crossrail, Thameslink).

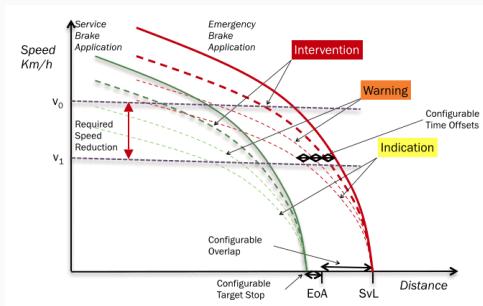


Trains are **hybrid systems** (discrete and continuous behaviour)

# Cyber-Physical Railway Signalling Systems

Trains are **hybrid systems** (discrete and continuous behaviour)<sup>3</sup>.

- European Vital Computed (EVC) computes braking curves and intervenes if braking curves are breached.



<sup>3</sup> <https://www.graffica.co.uk/case-studies/hermes-etcs-modelling/>



# Formal Methods for Railway Signalling Systems

Formal methods have been used in the railway domain, for example:

- The **B** method Paris Metro, Paris Roissy Airport shuttle.

Formal Verification of **control tables** and interlocking software (Solid State Interlocking (SSI)):

- push-button model-checking approaches.

ROUTE		INTERLOCKING				CONTROL				
		REQUIRES	SET & LOCKS POINTS		REQUIRES	ASPECT	SIGNAL AHEAD	REQUIRES TO CLEAR		
From	To	ROUTE NORMAL	NORMAL	REVERSE	KEYLOCK			NORMAL	CLEAR	AT TIME OF CLEARING ONLY TO CLEAR OR
1	3						Y G	3 AT R# 3 AT Y# OR G#		
2	4						Y G	4 AT R# 4 AT Y# OR G#		
3(1)	23	16,24,4(1),4(2),3(2)		103,104	201,202, 203,204	Y+JL	23 AT R#	37,9T,103T,24T,41T, 23T,104T,8T,4T	42	42 FOR 60 sec
3(2)	15	16,24,4(1),4(2),3(1)	103,104			Y	15 AT R# 15 AT G#	37,9T,103T,16T,42T, 15T,104T,8T,4T	41	41 FOR 60 sec
4(1)	16	15,23,3(1),3(2),4(2)	104,103			Y	16 AT R# 16 AT G#	47,9T,104T,15T,42T, 16T,103T,9T,3T	41	41 FOR 60 sec
4(2)	24	15,23,3(1),3(2),4(1)		104,103	201,202, 203,204	Y+JL	24 AT R#	47,9T,104T,23T,41T, 24T,103T,9T,3T	42	42 FOR 60 sec
15	UP BLOCK SECTION	23,4(1),4(2)	104,103			G		15T,104T,8T,4T,2T,TOL		
23	UP BLOCK SECTION	15,4(1),4(2)		104,103		G		23T,104T,8T,4T,2T,TOL		
16	DOWN BLOCK SECTION	24,3(1),3(2)	103,104			G		16T,103T,9T,3T,1T,TOL		
24	DOWN BLOCK SECTION	16,3(1),3(2)		103,104		G		24T,103T,9T,3T,1T,TOL		

<sup>3</sup> Control Table example from S. Vanit-Anunchai: Verification of Railway Interlocking Tables Using Coloured Petri Nets. COORDINATION, 2010.

# Formal Methods for Railway Signalling Systems

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- The **B** method Paris Metro, Paris Roissy Airport shuttle.

Formal Verification of control tables and the **interlocking software** (Solid State Interlocking (SSI)):

- automated theorem provers (e.g., The Formal Route company).

```
*QR117B(M) / route request block for route R117B(M)
  if R117B(M) a / route R117B(M) is available
    USD-CA f, OSC-BA f, OSV-BA f / sub-route and sub-overlaps are free
  then if OSL-AC l, / sub-overlap is OSL-AC locked
    P223 fr, P224 fr / points P223, P224 free to move reverse
  then @P223QR \ / call subroutine P223QR
  if OSD-BC f / sub-overlap is OSD-BC is free
    LTR04 xs / latch (boolean flag) not set (false)
    P224 cr / point P224 commanded reverse or free to move reverse
  then R117B(M) s / set route set flag for R117B(M)
    USD-AC l, USC-AB l, USB-AB l, USA-AB l / set sub-routes/overlaps
    P224 cr / command point P224 reverse
    LARR xs / clear latch LARR
    S117 clear bpull / clear signal button pull flag
    if P223 xcr, P223 rf then / check point states
      @P223QR / point command subroutine
      EP230 = 0 \ / reset timer EP230
```

<sup>3</sup>SSI example from Iliasov et al.: Formal Verification of Signalling Programs with SafeCap. SAFECOMP, 2018.

# Framework for CPS Design and Analysis

**Formal** CPS development framework which utilises abstraction and refinement.

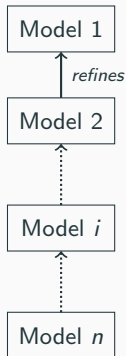
Enables a **multifaceted** CPS design:

- simulation-based system validation and analysis,
- model constraints and safe parameter values via reachability analysis.

Improves **scalability** of formal verification:

- automation of formal verification of hybrid systems,
- challenge of deriving differential invariant.

State-based pivot model  
(A)



# Framework for CPS Design and Analysis

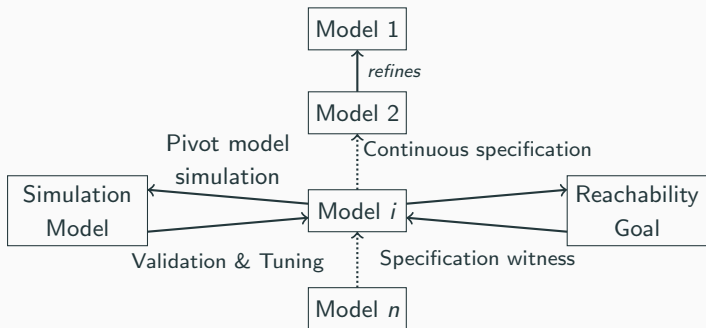
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Simulation-based analysis  
(C)

State-based pivot model  
(A)

Reachability analysis  
(B)



# Framework for CPS Design and Analysis

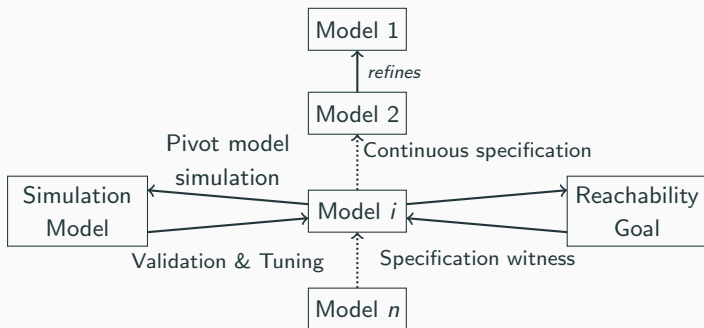
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Simulation-based analysis  
(C)

State-based pivot model  
(A)

Reachability analysis  
(B)



## **From Event-B to Hybridised Event-B**

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# From Event-B to Hybridised Event-B

The **B** method:

- formal software development method proposed by J.-R. Abrial.

The **Event-B** method:

- evolution of the **B** method for formal system-level modelling and verification.
- key features of the **Event-B** method:
  - set-theoretic modelling notation,
  - refinement- and proof- driven approach,
  - good tool support (Eclipse-based Rodin platform, ProB model checker, Theory plug-in, SMT solvers).

Both methods are used in academia and industry (e.g., Siemens Transportation, ALSTOM, CLEARSY and others)

# From Event-B to Hybrid Event-B

The structure of **Event-B** models:

- a *context* holds static information about the system,
- a *machine* describes dynamic system aspects,
- properties about the system can be expressed as invariants (e.g.  $\text{inv}_2$ ),
- 10 different types of possible proof obligations,
- (discrete) **Event-B** model verification automation has been significantly improved.

**CONTEXT** ctx0

**SETS**

CRS

**CONSTANTS**

m

**AXIOMS**

axm<sub>0</sub> finite(CRS)

axm<sub>0</sub>  $m \in \mathbb{N}1$

axm<sub>0</sub>  $m \leq \text{card}(\text{CRS})$

**END**



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**MACHINE** m0

**VARIABLES**

x

**INVARIANTS**

$\text{inv}_1 \quad x \in \mathbb{N}$

**inv<sub>2</sub>**  $x \leq 11$

**EVENTS**

INITIALISATION

**THEN**

act<sub>1</sub> :  $x := 0$

**END**

Increment

**WHERE**

grd<sub>1</sub> :  $x \leq 10$

**THEN**

act<sub>1</sub> :  $x := x + 1$

**END**

**END**

# From Event-B to Hybrid Event-B

**MACHINE** m0

**VARIABLES**

x

**INVARIANTS**

inv<sub>1</sub> x ∈ ℕ

inv<sub>2</sub> x ≤ 11

**EVENTS**

INITIALISATION

**THEN**

act<sub>1</sub> : x := 0

**END**

Increment

**WHERE**

grd<sub>1</sub> : x ≤ 10

**THEN**

act<sub>1</sub> : x := x + 1

**END**

**END**

## Invariant Preservation Rule

Axioms

Invariants

Event Guards

Event BAP

⊢

Modified Specific Invariant

x ∈ ℕ

x ≤ 10

⊢

x + 1 ≤ 11

x ∈ ℕ

x = 0

⊢

x ≤ 11

# From Event-B to Hybrid Event-B

**MACHINE** m0

**VARIABLES**

x

**INVARIANTS**

inv<sub>1</sub> x ∈ ℕ

inv<sub>2</sub> x ≤ 11

**EVENTS**

INITIALISATION

**THEN**

act<sub>1</sub> : x := 0

**END**

Increment

**WHERE**

grd<sub>1</sub> : ⊤

**THEN**

act<sub>1</sub> : x : | x' = x + 1 ∧ x' + 1 ≤ 11

**END**

**END**

## Feasibility

Axioms

Invariants

Event Guards

⊢

∃v' · Event BAP

Note: Rewriting act<sub>1</sub> with *such that* and strengthening before-after predicate we can automatically prove inv<sub>2</sub> but need to prove feasibility.

# From Event-B to Hybridised Event-B

The Rodin Theory plug-in allows extending the **Event-B** mathematical language:<sup>4</sup>

**THEORY** Seq

**TYPE PARAMETERS** A

**OPERATORS**

*seq* expression  $seq(a : \mathbb{P}(A))$

**direct definition**

$seq(a : \mathbb{P}(A)) \triangleq \{n, f \cdot n \in \mathbb{N} \wedge f \in 1..n \rightarrow a|f\}$

⋮

**AXIOMS**

*seqIsFinite*  $\forall s, a \cdot a \subseteq A \wedge s \in seq(a) \Rightarrow finite(s)$

⋮

**PROOF RULES**

⋮

**END**

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<sup>4</sup>Event-B theory example based on  
[https://wiki.event-b.org/index.php/Theory\\_Plug-in](https://wiki.event-b.org/index.php/Theory_Plug-in)

# From Event-B to Hybridised Event-B

Hybrid systems are dynamical systems that exhibit discrete and continuous behaviour:

- a hybrid automaton model is used for describing hybrid systems.

The **Event-B** method for hybrid systems:

- Banach et al. Hybrid Event-B: Core Hybrid Event-B I: Single Hybrid Event-B machines
  - new *pliant* events for continuous actions,
  - approach is not tool supported.
- Dupont et al. Correct-by-Construction Design of Hybrid Systems Based on Refinement and Proof (PhD thesis)
  - new Event-B theories (Reals, continuous functions, differential equations, theory of approximations),
  - hybrid system modelling and refinement patterns (generic hybrid Event-B model).

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# From Event-B to Hybridised Event-B

**THEORY** DiffEq **IMPORT** Functions

**TYPE PARAMETERS**  $E, F$

**DATATYPES**

**DE**( $F$ ) **constructors**  $\text{ode}(f, \eta_0, t_0), \dots$

**OPERATORS**

**solutionOf** *predicate* ( $D : \mathbb{P}(\mathbb{R}), \eta : \mathbb{R} \rightarrow F, \mathcal{E} : \mathbf{DE}(F)$ ) ...

**Solvable** *predicate* ( $D : \mathbb{P}(\mathbb{R}), \mathcal{E} : \mathbf{DE}(F)$ ) ...

**CBAP** *predicate* ( $t, t' : \mathbb{R}^+, x_p, x'_p : \mathbb{R} \rightarrow F, \mathcal{P} : \mathbb{P}((\mathbb{R} \rightarrow F) \times (\mathbb{R} \rightarrow F)), H : \mathbb{P}(F)$ )

...

$:\sim$  *predicate* ( $t, t' : \mathbb{R}^+, x_p, x'_p : \mathbb{R} \rightarrow F, \mathcal{E} : \mathbf{DE}(F), H : \mathbb{P}(F)$ )

**well-definedness condition** **Solvable**( $[t, t'], \mathcal{E}$ )

**direct definition** **solutionOf**( $[t, t'], x'_p, \mathcal{E}$ )  $\wedge \dots$

...

**AXIOMS**

CauchyLipschitz: — *external*

$\forall \mathcal{E}, D, D_F \cdot \mathcal{E} \in \mathbf{DE}(F) \wedge \dots \Rightarrow \mathbf{Solvable}(D, \mathcal{E})$

...

- use of theories to integrate continuous features  
 $\Rightarrow$  e.g. *continuous behaviour using differential equations*
- exploit WD to ensure the correct use of operators/theorems

# From Event-B to Hybridised Event-B

Continuous state variables = *functions of time* ( $\in \mathbb{R} \mapsto S$ )

$\Rightarrow$  **continuous evolution** as CBAP

$$\text{CBAP}(t, t', x_p, x'_p, \mathcal{P}, H) \equiv$$

$$x_p : |_{t \rightarrow t'} \mathcal{P}(x_p, x'_p) \& H \equiv$$

$$[0, t[ \triangleleft x'_p = [0, t[ \triangleleft x_p \quad (\text{Past Preservation})$$

$$\wedge \mathcal{P}([0, t[ \triangleleft x_p, [t, t'] \triangleleft x'_p) \quad (\text{Predicate})$$

$$\wedge \forall t^* \in [t, t'], x_p(t^*) \in H \quad (\text{Evolution Dom.})$$

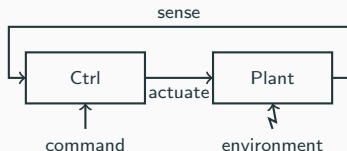
**Note:** shorthand for differential equations:

$$x_p : \sim_{t \rightarrow t'} \mathcal{E} \& H \equiv x_p : |_{t \rightarrow t'} \text{solutionOf}([t, t'], \mathcal{E}, x'_p) \& H$$



# From Event-B to Hybridised Event-B

Hybridised **Event-B** patterns formalise a generic controller-plant-loop hybrid system as Event-B model:



Hybridised **Event-B** machine modelling pattern:

**MACHINE** Generic

**EXTENDS** *DiffEquations*

**VARIABLES**  $t, x_s, x_p$

**INVARIANTS**

$inv_1: t \in \mathbb{R}^+$

$inv_2: x_s \in \text{STATES}$

$inv_3: x_p \in \mathbb{R} \leftrightarrow S$

$inv_4: [0, t] \subseteq \text{dom}(x_p)$

- use developed theories (e.g., differential equations),
- explicit time ( $t$ ),
- discrete state ( $x_s$ ) + continuous state ( $x_p$ , function of time).

# From Event-B to Hybridised Event-B

Generic events of hybridised **Event-B** modelling pattern:

Actuate

**ANY**  $\mathcal{P}, s, H, t'$

**WHERE**

$\text{grd}_0: t' > t$

$\text{grd}_1: \mathcal{P} \in (\mathbb{R}^+ \rightarrow S) \times (\mathbb{R}^+ \rightarrow S)$

$\text{grd}_2: \text{Feasible}([t, t'], x_p, \mathcal{P}, H)$

$\text{grd}_3: s \subseteq \text{STATES} \wedge x_s \in s$

$\text{grd}_4: H \subseteq S \wedge x_p(t) \in H$

**THEN**

$\text{act}_1: x_p :|_{t \rightarrow t'} \mathcal{P}(x_p, x'_p) \ \& \ H$

**END**

Sense

**ANY**  $s, p$

**WHERE**

$\text{grd}_1: s \in \mathbb{P}1(\text{STATES})$

$\text{grd}_2: p \in \mathbb{P}(\text{STATES} \times \mathbb{R} \times S)$

$\text{grd}_3: (x_s \mapsto t \mapsto x_p(t)) \in p$

**THEN**

$\text{act}_1: x_s : \in s$

**END**

- discrete event **Sense** + continuous event **Actuate** (passing of time),
- **Actuate** based on **CBAP**, WD in guard (proved in refinement with guard strengthening),
- Additional generic events **Behave** and **Transition** model changes induced by environment and user.

# From Event-B to Hybridised Event-B

New types of proof obligations:

- Continuous invariant preservation: if the invariant is true on  $[0, t]$ , then it must be true on  $[t, t']$ , i.e., on the whole duration of the continuous event:

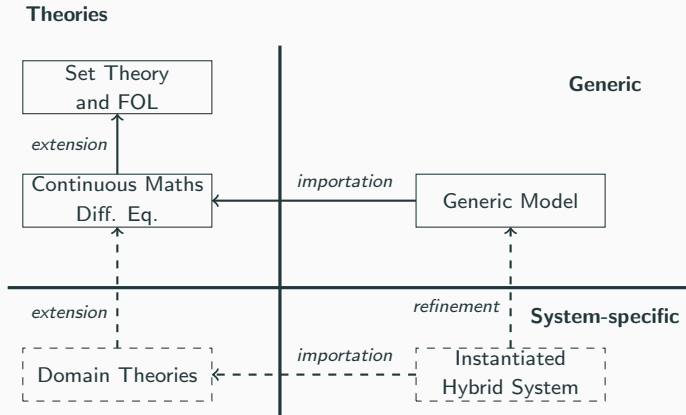
$$\Gamma, \mathcal{I}([0, t] \triangleleft x_p), \text{CBAP}(t, t', x_p, x'_p, \mathcal{P}, \mathcal{H}) \vdash \mathcal{I}([t, t'] \triangleleft x'_p) \quad (\text{CINV})$$

- Continuous feasibility requires to prove that, if the event is triggered, then its action can be performed:

$$\Gamma \vdash \exists t' \cdot t' \in \mathbb{R}^+ \wedge t' > t \wedge \mathbf{Feasible}([t, t'], x_p, \mathcal{P}, \mathcal{H}_{saf}) \quad (\text{CFIS})$$

**Important:** Proof-obligations related to continuous system behaviour of the model are generally complex and proved interactively.

# Hybridised Event-B for CPS Design Framework



The following slides present the framework application for developing a cyber-physical railway signalling system.

# Cyber-Physical Railway Signalling System: Speed Controller

1<sup>st</sup> refinement of the generic introduces rolling stock.

- A driver (or ATO system) controls a train engine power (tractive force) -  $f$  - which yields an acceleration,
- Davis Resistance equation in Equation (1), where  $A, B, C$  are fixed parameters and  $v(t)$  is the speed of a train at time  $t$ :

$$\begin{cases} \dot{v}(t) &= \pm(f - (A + B \cdot v(t) + C \cdot v(t)^2))/M_{train} \\ \dot{p}(t) &= v(t) \end{cases} \quad (1)$$

- The hybrid automaton model of the train speed controller:



- Properties of the train are gathered in the Train *domain theory*,
- This theory mainly defines the *Davis equation* and its properties

**THEORY** Trains

**OPERATORS**

DavisResistance *expression*  $(a : \mathbb{R}, b : \mathbb{R}, c : \mathbb{R})$

**well-definedness condition**  $a \geq 0, b \geq 0, c \geq 0$

**direct definition**  $(\lambda v \cdot v \in \mathbb{R} \mid a + bv + cv^2)$

...

**THEOREMS**

...

**END**

The context defines the constants of the system:

- Davis coefficients ( $a, b, c$ ), traction power limits ( $f_{min}, f_{max}$ )

Also, the context introduces the stopping distance function **StopDist** and controller models.

**CONTEXT** TrainCtx

**CONSTANTS**

free\_move, restricted\_move

StopDist

$a, b, c, f_{min}, f_{max}, f_{dec\_min}$

**AXIOMS**

axm<sub>1</sub>:  $a, b, c \in \mathbb{R}^+$

axm<sub>2</sub>:  $f_{min}, f_{max}, f_{dec\_min} \in \mathbb{R}$

axm<sub>3</sub>:  $\text{StopDist} \in (\mathbb{R} \times \mathbb{R}^+) \rightarrow \mathbb{R}^+$

axm<sub>5</sub>:  $\text{partition}(\text{STATES}, \{\text{free\_move}\}, \{\text{restricted\_move}\})$

...

**MACHINE** TrainMach **REFINES** Generic

**VARIABLES**  $t, x_{st}, tp, tv, ta, f, \text{EoA}$

**INVARIANTS**

$$\text{inv}_1: tp, tv, ta \in \mathbb{R} \leftrightarrow \mathbb{R}$$

$$\text{inv}_2: [0, t] \subseteq \text{dom}(tp), \dots$$

$$\text{inv}_3: \text{EoA} \in \mathbb{R}^+$$

$$\text{inv}_4: f_{\min} \leq f \wedge f \leq f_{\max}$$

$$\text{inv}_5: x_p = [ta \ tv \ tp]^T$$

$$\text{saf}_1: \forall t^* \cdot t^* \in [0, t] \Rightarrow tp(t^*) \leq \text{EoA}$$

$$\text{phy}_1: \forall t^* \cdot t^* \in [0, t] \Rightarrow tv(t^*) \geq 0$$

Safety property as: **at all times the train must remain within the issued movement authority:**

- expressed as Event-B invariant  $\text{saf}_1$ ,
- an additional physics property  $\text{phy}_1$ .



Sense\_to\_restricted

**REFINES** Sense

**WHERE**

$\text{grd}_1: tp(t) + \text{StopDist}(ta(t) \mapsto tv(t)) \geq \text{EoA}$

**WITH**

$st: st = \{\text{restricted\_move}\}$

$p: p = \text{STATES} \times \mathbb{R} \times \{v^* \mapsto p^* \mid p^* + \text{StopDist}(f_{dec\_min} \mapsto v^*) \geq \text{EoA}\}$

**THEN**

$\text{act}_1: x_{st} := \text{restricted\_move}$

**END**

# Cyber-Physical Railway Signalling System: Speed Controller

Actuate\_move **REFINES** Actuate

**ANY**  $t'$

**WHERE**

$\text{grd}_1: tp(t) + \text{StopDist}(ta(t) \mapsto tv(t)) \leq \text{EoA}$

$\text{grd}_2: t < t'$

**WITH**

$x'_p: x'_p = [ta \ tv \ tp]^\top$

$\mathcal{P}: \mathcal{P} = \dots$

$H: H = \dots$

$st: st = \text{STATES}$

**THEN**

$\text{act}_1: ta, tv, tp: |_{t \rightarrow t'}$

**solutionOf** $([t, t'], [tv \ tp]^\top, \text{DavisEquation}(a, b, c, f, t, tv(t), tp(t))) \wedge$

$ta = tv$

$\&tp + \text{StopDist}(ta \mapsto tv) \leq \text{EoA} \wedge tv \geq 0$

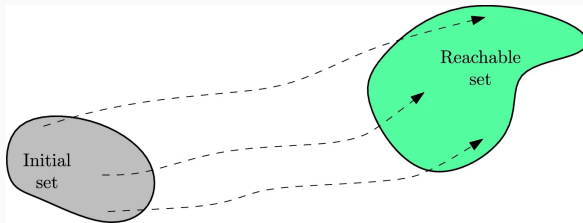
**END**

# Cyber-Physical Railway Signalling System: Proof Statistic

Refinement	PO Type	POs	Auto.	Inter.
<b>Speed Controller</b>		55	36	19
	WD	12	12	0
	GRD	11	11	0
	<b>INV</b>	18	10	<b>8</b>
	<b>FIS</b>	8	0	<b>8</b>
	SIM	6	3	3
Communication		85	71	14
	WD	31	31	0
	GRD	12	7	5
	INV	42	33	9
	FIS	0	0	0
	SIM	0	0	0
Total		140	119	21

# Reachability Analysis for Hybridised Event-B

Can **reachability analysis** help to address verification automation challenges of hybridised Event-B models (similar to how ProB model checker is used for discrete systems)?



Computing reachable states of a **hybrid automaton** requires computing *runs* of the hybrid system.

Reachability enabled verification **tactic** of **CINV**:

1. Strengthen actuation events actions such that  $H \subseteq \mathcal{I}$ ,
2. Generating proof-obligation (automatically),
  - 2 CFIS proof obligations were generated (for the free and restricted modes).
3. Translate proof-obligations to reachability analysis tool (JuliaReach, manually),
  - translate other related functions - *StopDist*.
4. Define initial values  $\mathcal{X}_0$  for the reachability problem,
5. Compute and check solution produced reachability tool,
  - check existence of an interval  $[0, t']$  for which reachset  $\mathcal{R}$  of continuous  $x_p$  with initial values  $\mathcal{X}_0$  satisfies a strengthened local invariant  $\mathcal{H}$ .

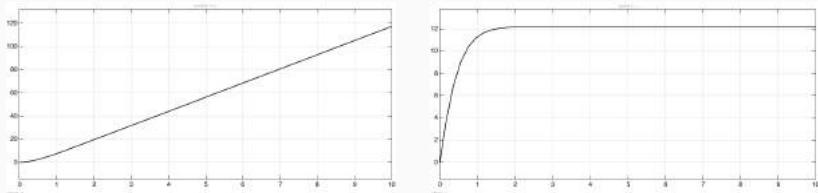
# Cyber-Physical Railway Signalling System: Proof Statistic

Refinement	PO Type	POs	Auto.	Inter.
<b>Speed Controller</b>		55	36 (48)	19 (7)
	WD	12	12	0
	GRD	11	11	0
	INV	18	10 (14)	8 (4)
	FIS	8	0 (8)	8 (0)
	SIM	6	3	3
Communication		85	71	14
	WD	31	31	0
	GRD	12	7	5
	INV	42	33	9
	FIS	0	0	0
	SIM	0	0	0
Total		140	119	21

# Cyber-Physical Railway Signalling System: Validation

To enable model animation and validation we aim to connect hybridised **Event-B** with Simulink/Stateflow.

To validate the speed controller model we (manually) translated it to Simulink/Stateflow.



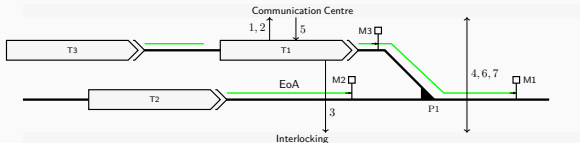
**Figure 4:** TGV train simulation with Davis equation coefficients for TGV:  
 $a = 25$ ,  $b = 1.188$  and  $c = 0.0703728$

# Cyber-Physical Railway Signalling System: Other Sub-Systems

2<sup>nd</sup> refinement introduces other sub-systems of the signalling system:

- communication centres, interlocking and infrastructure,
- communication protocol.

The **generic** railway signalling is based on ETCS Level 3 and CBTC systems.



Communication protocol was modelled by using developed **Event-B** communication modelling patterns.

**To formally demonstrate** that the generic signalling system issues **safe** movement authority and ensures safe point crossing.



# Cyber-Physical Railway Signalling System: Proof Statistic

Refinement	PO Type	POs	Auto.	Inter.
<b>Speed Controller</b>		55	36 (48)	19 (7)
	WD	12	12	0
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	FIS	0	0	0
	SIM	0	0	0
Total		140	119	21

# Conclusions and Next Steps

## **In summary:**

- The complexity of developing complex CPS can be reduced by using refinement and abstraction.
- Our proposed framework provides a more comprehensive formal CPS development.
- Reachability analysis can help to improve verification automation of hybridised Event-B models.

## **Next steps in the short-term:**

- Facilitate an automatic translation of hybridised Event-B models to JuliaReach,
- develop new Event-B theories.

## (Long-term) Future Work

Explore synergies between proof and reachability analysis for CPS system verification and code generation:

- proving single CINV/CFIS proof-obligations (still many open questions),
- proving CPS Event-B sub-models,
- discovering model constraints and safe parameter values,
- discretisation of continuous model and code generation (discovering  $t'$ ).

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**Website with CfP:** <https://www.irit.fr/FE-CPS-2023/>

**Invited talks:** Ana Cavalcanti (University of York, UK) and Claudio Gomes (Aarhus University, Denmark)

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