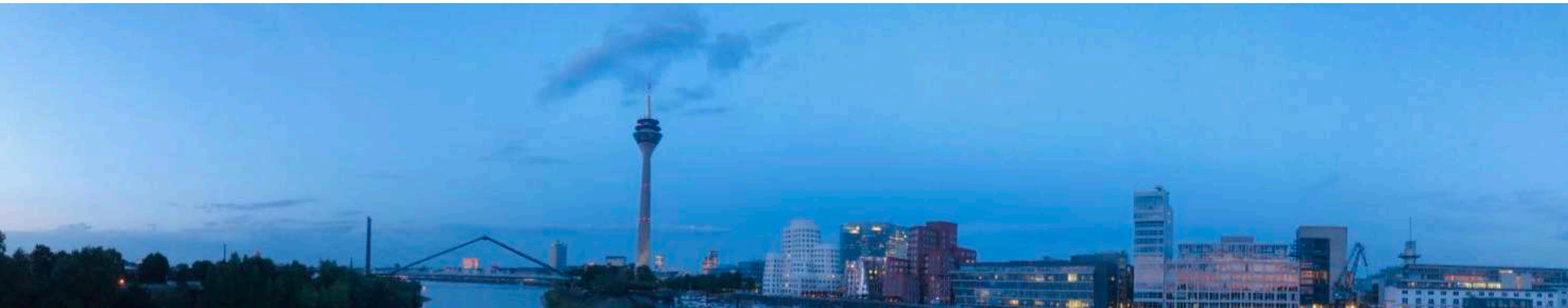


New Ways of Using Formal Models in Industry

Michael Leuschel



Overview

- Part 1: Overview of 25 years of history of B
 - taken from FMICS 2020 article with Michael Butler, Philipp Körner, Sebastian Krings, Thierry Lecomte, Luis-Fernando Mejia, Laurent Voisin entitled “The First Twenty-Five Years of Industrial Use of the B-Method”
- Part 2: Commandments and Lessons using B and building tools for B, Z, and other formal methods (mainly the tool ProB <https://prob.hhu.de>) inspired by Bowen, Hall, Hinchey

Part 1: History

Formal Methods

- Mathematical techniques to produce correct software and systems
- Highly recommended e.g. for SIL3/SIL4 railway applications (CENELEC)
- Some Benefits: Problems detected earlier and correction less costly, lower level testing (unit) not required as SW execution errors proven impossible

B Formal Method



Specification



Tool Support

Refinement

Origins of B

- Train protection system SACEM for Paris RER Line A
 - put into operation in 1988, sketch of the B-Method by Jean-Raymond Abrial
- 1989 project by Alstom, RATP, SNCF to develop tools and train engineers
- Paris Metro Line 14 contract won by Matra Transport (now Siemens Transportation Systems)
 - **1995:** B tools industrialised by Digilog (then Steria, now CLEARSY) leading to Atelier-B
 - ready by end 1998: 110 kLOC B model, 83% automatic proof, 86 kLOC Ada
 - Still in version 1.0, no single issue caused by software



B Logical Foundations



- Typed **first-order predicate logic** with equality
 - Well-Definedness Conditions to stay in two-valued logic
 - **Arithmetic** over mathematical integers and implementable integers
 - **Set theory**
 - Sets, Relations, Functions, Sequences
 - including **higher-order** functions
 - B is simpler than its predecessor Z
 - and provides structuring and refinement for proving and code generation
- related state-based formal methods:
Z, TLA+, Alloy, VDM, ASM

$$p \in \text{dom}(a) \rightarrow \text{dom}(a) \wedge \forall i \cdot (i \in 1..(\text{size}(a)-1) \Rightarrow p(a(i)) < p(a(i+1)))$$

B Structuring

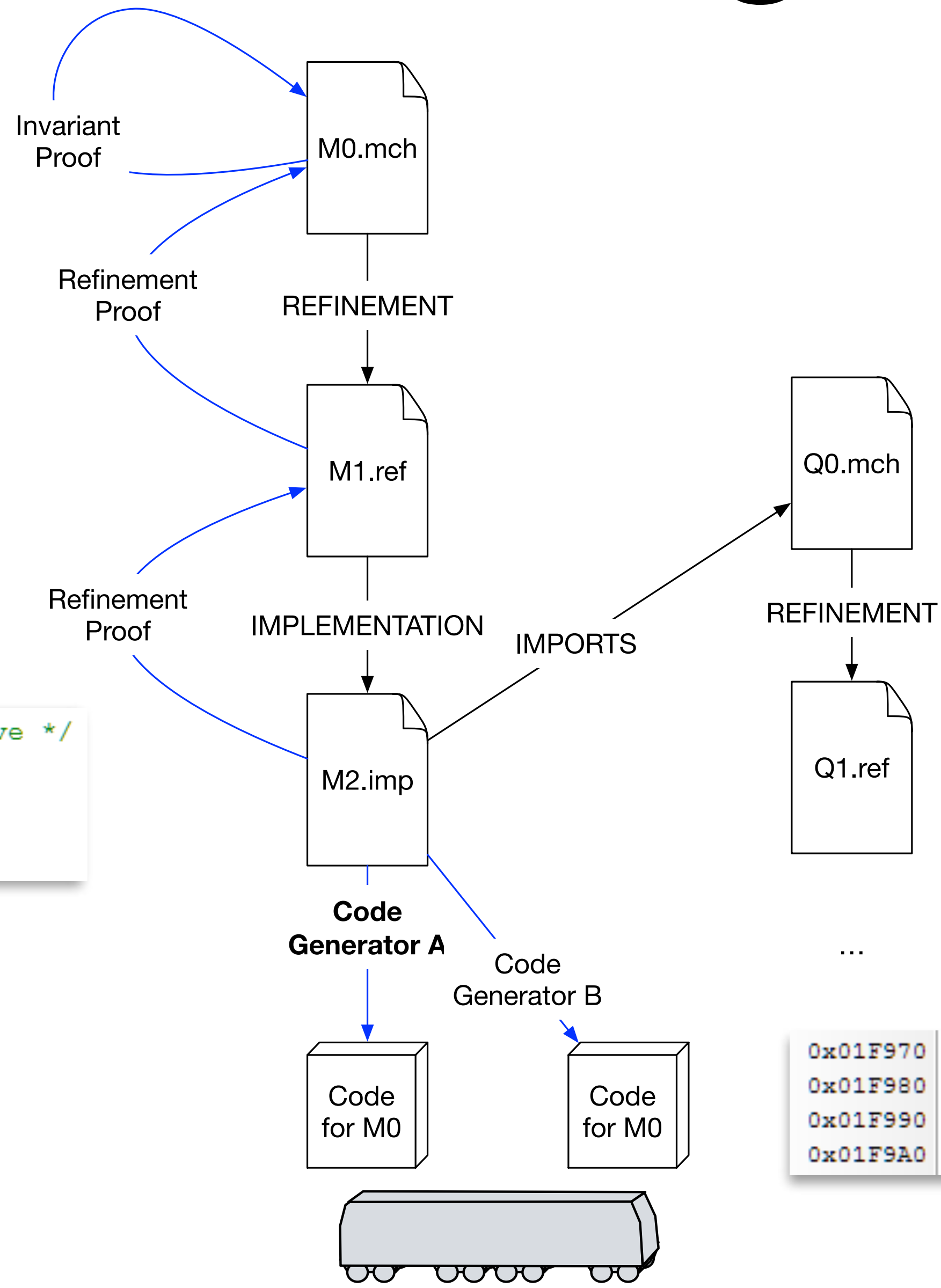


- Enables **decomposing** a specification
- Ensures that code generated for a B machine can be safely re-used
- Ensures **tractable proof** obligations
- Some key concepts:
 - VARIABLES vs CONSTANTS and associated INVARIANTS and PROPERTIES
 - OPERATIONS to modify variable values
 - Various B machine structuring mechanisms (INCLUDES, USES, SEES, ...)
 - REFINEMENT and IMPLEMENTATION machines

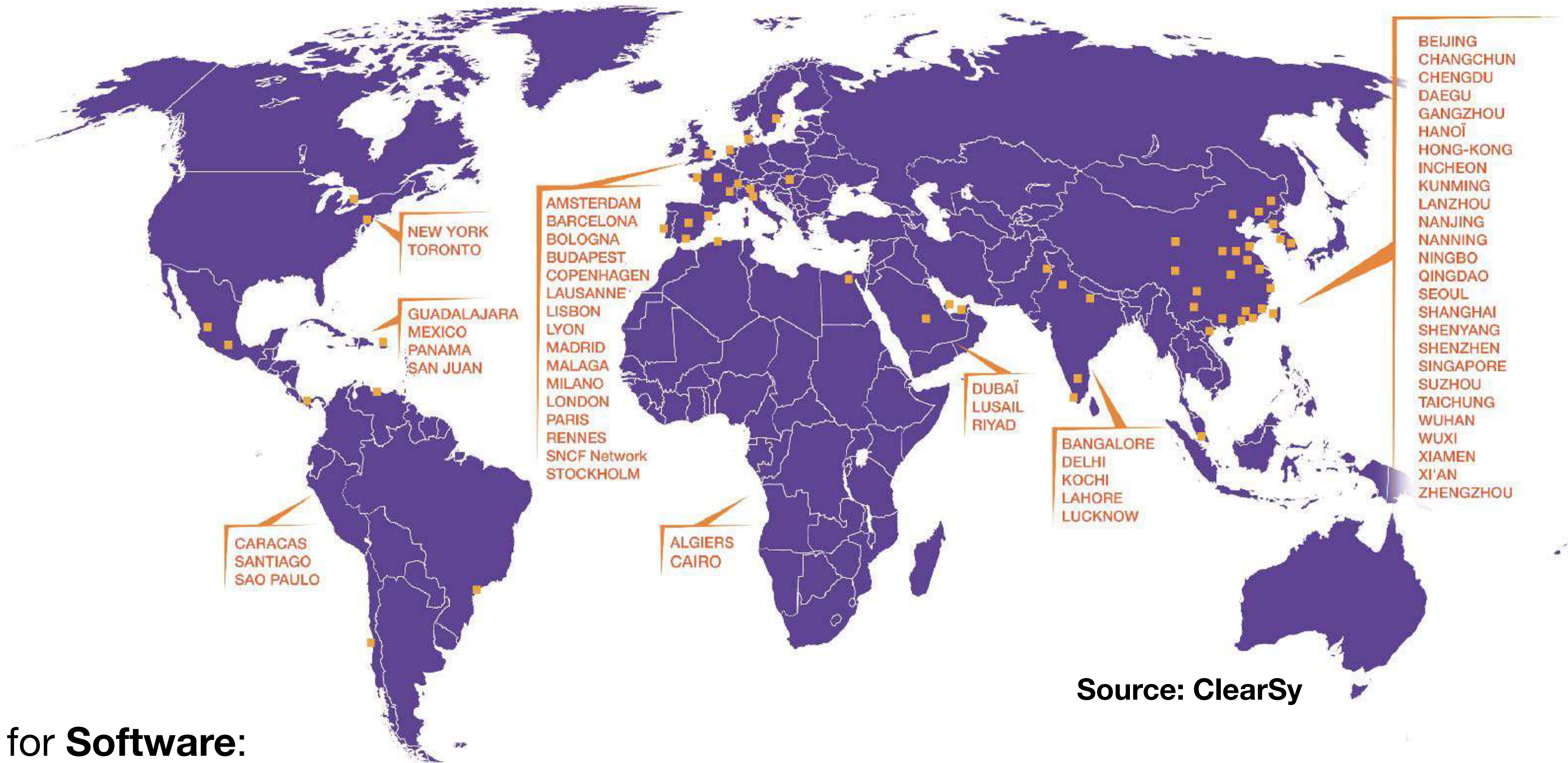
B Structuring

```
activation_sequence = /* Activation d'une séquence non active */  
PRE ¬(sequences = sequences_actives) THEN  
  ANY sequ WHERE  
    sequ ∈ sequences - sequences_actives  
  THEN  
    sequences_actives := sequences_actives U {sequ}  
  END  
END;  
END;
```

```
activation_sequence = /* Activation d'une séquence non active */  
VAR sequ IN  
  sequ <-- indexSequenceInactive;  
  activeSequence(sequ)  
END;  
END;
```



0x01F970	FFFF	8B4C	2440	89C5	8D7D	0C8B	4110	89CE
0x01F980	83C6	0C8D	1485	0000	0000	8D42	0883	F807
0x01F990	7617	F7C7	0400	0000	740F	8B41	0C8D	7D10
0x01F9A0	83C6	0489	450C	8D42	04FC	89C1	C1E9	02F3



- B for **Software**:

- about 30% of CBTC systems worldwide employ the B formal method
- Urbalis 400, Alstom, over 100 metro lines worldwide, 25% of worldwide CBTC market



One citation

- “Beyond the technological challenge of using such a complex formal method in an industrial context, it is now clear for us that building software using **B is not more expensive** than using conventional methods. Better, due to our experience in using this method, we can assert that using **B is cheaper when considering the whole development process** (from specification to validation and sometimes certification)”
- From: Didier Essamé, Daniel Dollé: B in Large-Scale Projects: The Canarsie Line CBTC Experience. In: LNCS Vol. 4355, Springer.

B for System Modelling

Event-B for System Modelling

- Analyse an entire system of components
- Ensure that together they ensure safety (and functionality)
- Talks about **events** rather than **operations**
- Refinement
 - used to structure reasoning, view a system at different levels of granularity
 - requires a more liberal view of refinement

**First paper
on Event-B
already published
by Abrial in 1996**

Session 6: Chairman, M. Frappier (Univ. de Sherbrooke, Canada)

10h15 *Extending B without Changing it (for Developing Distributed Systems)*

Invited speaker: Jean-Raymond Abrial (Consultant independant, Paris, F)

in co-operation with départ. informatique de
l'Institut Universitaire de Technologie,
BUG and BIP

8th International Conference on:

"PUTTING INTO PRACTICE METHODS AND
TOOLS FOR INFORMATION SYSTEM DESIGN"

1st Conference on the B method

```
MAJUSCULE
RECHERCHER UNAL. SAGE
VARIABLES
# sage
INVARIANT
sage < 0. max. sage
INITIALISATION
sage := max. sage
OPERATIONS
revenir =
PRE
0 := sage
TREN
sage := sage + 1
END
Avec:
PRE
sage := max. sage
TREN
sage := sage + 1
END
END
```

November, 24-25-26, 1996
NANTES (France)

PROCEEDINGS

Editor: Henri HABRIAS

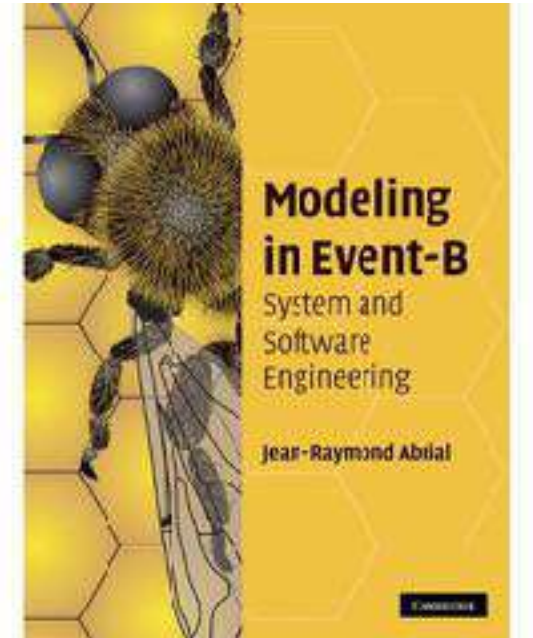
ISBN: 2-906082-25-2

Dépôt légal: Novembre 1996

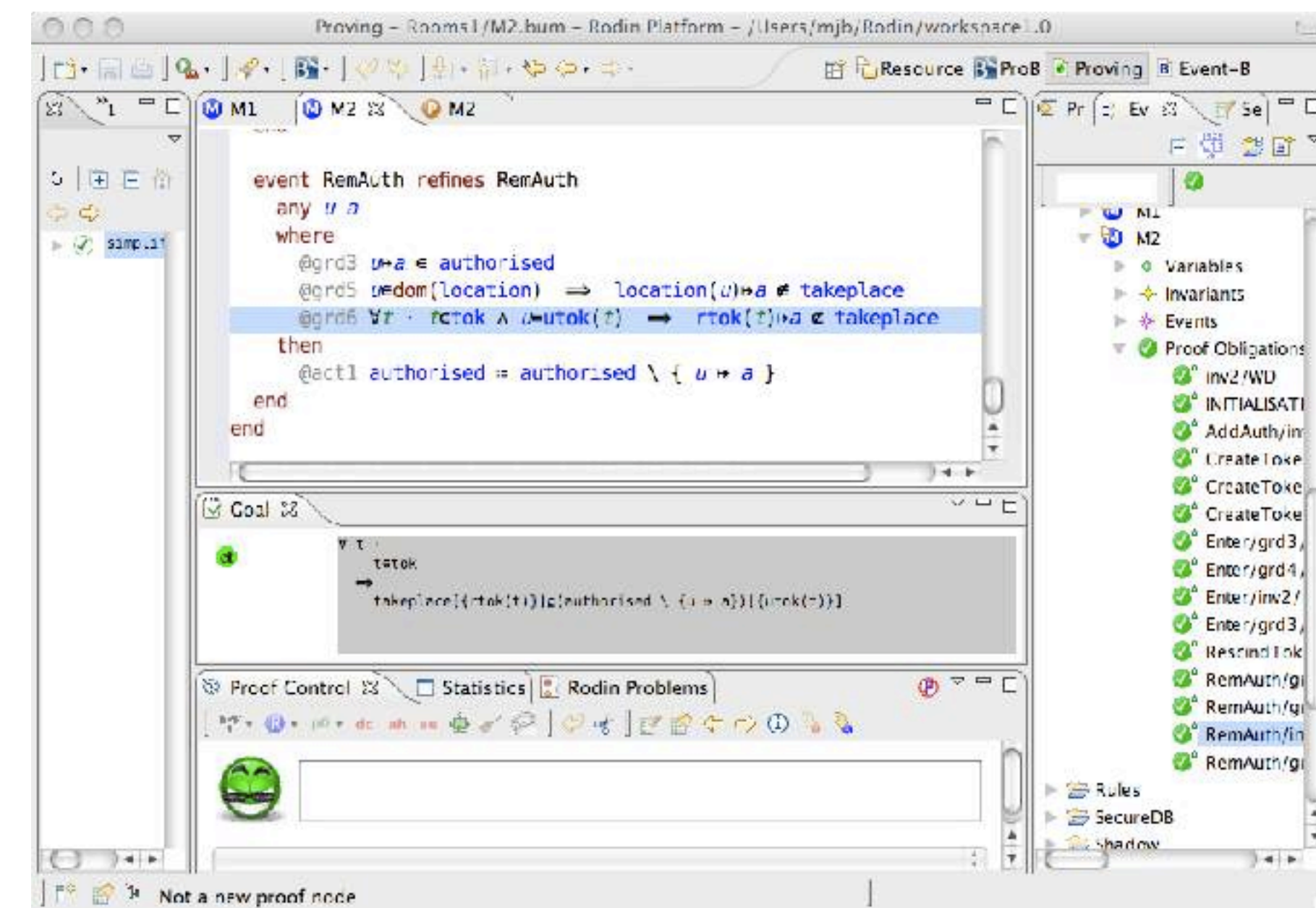
General Chairman: Henri HABRIAS,
IRIN-IUT de Nantes
3 Rue du Maréchal Joffre, BP 34103,
44041 NANTES Cedex 1 (France)
Tél. (02) 40 40 27 40 74



Foundations of Event-B



- Described in book “Modelling in Event-B” by Abrial (2010).
- Better proof automation thanks to simpler substitutions (aka statements) and proof obligations (witnesses, ...)
- More expressive and flexible refinement
- Some changes to expressions and predicates
- Foundations realised in the Rodin platform



Tool Support for Event-B

- Rodin
 - Eclipse-based IDE for POG, proof, ...
- Atelier B
 - also supports Event-B projects with POG and proof
- ProB
 - Multi-Level Animation, Visualization, Model Checking



CBTC models of ClearSy

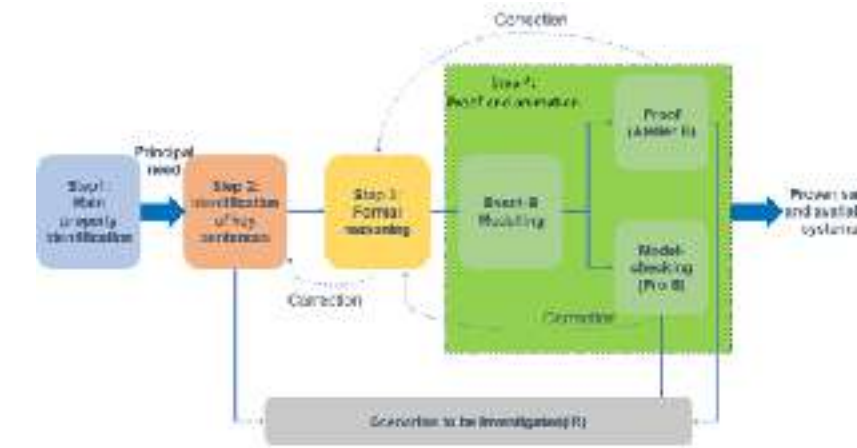


- Thales Toronto installed CBTC system for NYC Subway Line 7
- CBTC = Communication-based Train Control: automatic train control system using a combination of classical trackside train detection (TTD) and position reports sent by trains
- ClearSy was asked to perform a **formal** verification of the **safety** of the system for Thales Toronto
 - from November 2010 until December 2012
 - ClearSy was using the Event-B along with the Atelier-B prover

Summary of Results

- CBTC system **safety** (no collision, no derailment, no overspeeding, correct tracking,...) **formally verified** with B and Atelier-B
- **Key properties** and knowledge extracted and put into the formal model
- Formal **model** was **reused** for NYCT I2S [Sabatier, RSSR16], similar analyses have since then been carried out (Octys CBTC by RATP [Comptier et al., RSSR17],...)
- Showed that a **large** industrial, safety-critical system could be effectively formally analysed and proven correct using Event-B

Alstom Zone Controller



- System analysis carried out in 2018 for a large software system (CBTC Zone Controller) by Alstom, ClearSy and University of Düsseldorf [Comptier et al., RSSR'19]
- Software for this component generated using classical B
- Analysis with Atelier B and ProB
- Enabled to extract **key** safety **properties** which enable future evolution of the component

ETCS Hybrid Level 3

- Several formal methods case done of the ETCS Hybrid Level 3 (HL3) principles
- B Model and system developed by Thales and University of Düsseldorf
- Identified over 50 issues in various versions of the HL3 specification
- Formal model was used in **real-time** for field demonstration in December 2017 at ETCS National Integration Facility in Hitchin/UK, providing evidence that the HL3 principles are consistent and allow desired operational behaviour

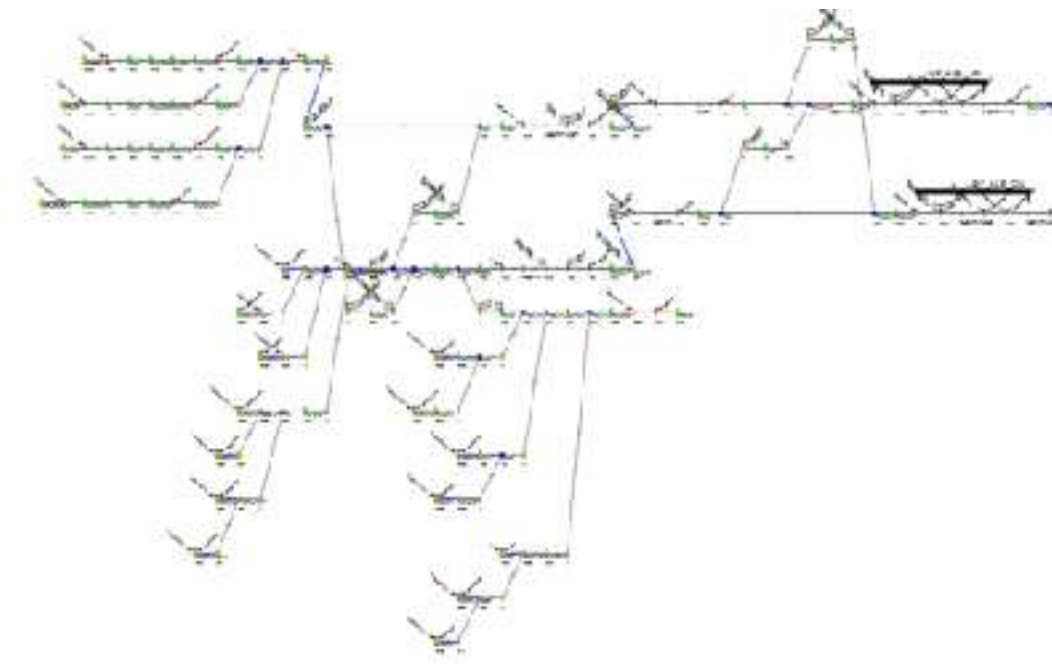
B for Data Validation

Data Validation

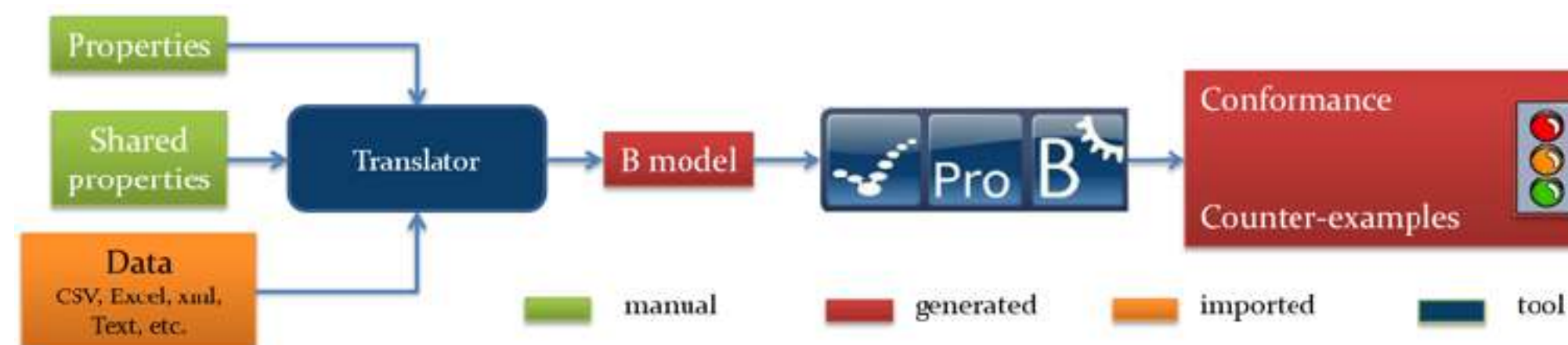
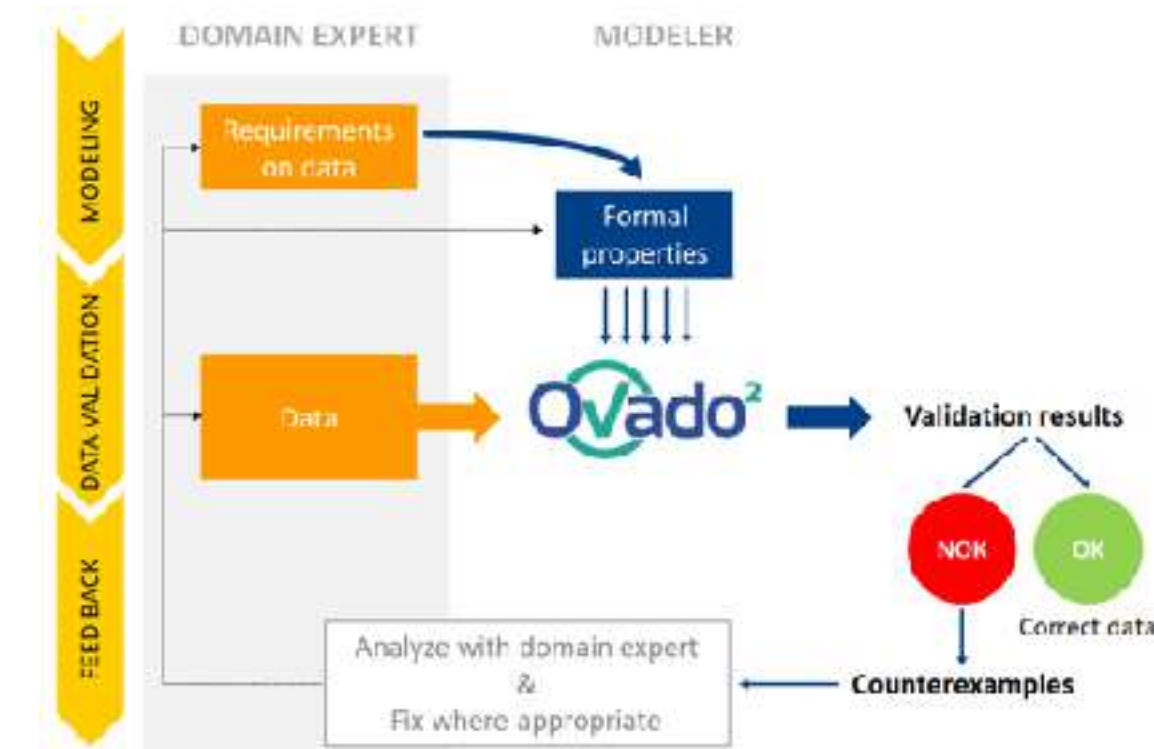
	A	B	C	D	E	F	G	H	I
1	Name	ID	IP	Type	UpLink	DownLink	Length	GPS 1	GPS 2
2	Route_tx_001	243		R	Route_tx_005	Route_vx_002	345		
3	Route_vx_002	128		R	Route_vx_002	EndLine_001	128		
4	Switch_w_003	236	192.16.4.55	S	Route_vx_128	Route_tx_005	23		
5	Relay_s_004	12	192.16.4.10	Y				N 50.85 963	O 6.84 201
6	Route_tx_005	3		R	Route_tx_005	Route_vx_128	291		
7	Relay_s_001	53	192.16.4.125	Y					
8	Route_tx_006	24		R	EndLine_001	Route_vx_002	110		
9	Route_vx_128	127		R	Route_tx_005	Route_vx_002	145		
10	Switch_w_009	212	192.16.4.10	S	Route_vx_128	Route_tx_005	31		
11	EndLine_003	0		L		Route_vx_002	1		
12	EndLine_001	1		F	Route_vx_002		1		
13	Signal_vx_002	32	192.16.4.12	G	Route_vx_128		22		
14	Signal_vx_003	33	192.16.4.13	G	Route_tx_005		21		
15	Relay_b_001	301		R	Route_vx_128			O N 50.85 933	O 6.84 508
16	Relay_b_002	302		R	Route_tx_005			O N 50.85 123	O 6.84 550

- What is the use of a formally proven software if some of its (non trivial) parameters are wrong ?
- Data Validation: Automatic check of large data sets against properties
- Properties : international standards, national regulations, manufacturer habits, customer requirements, safety assumptions made during development, ...
- E.g. metro line static data used by the automatic pilot (software) to drive safely

B for Data Validation

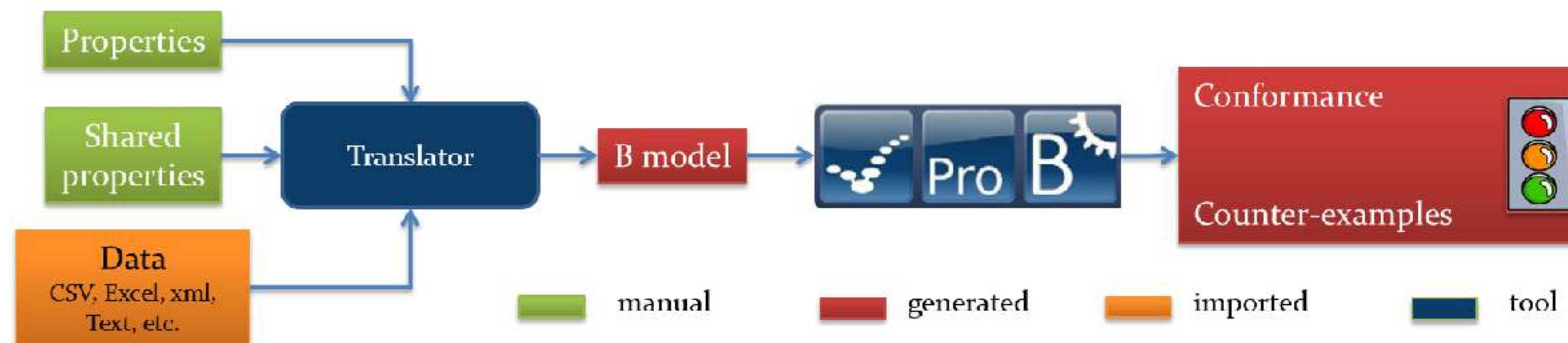


- Express properties in B: works well with graph-based properties or if software already developed with B
- Initial developments
 - OVADO for RATP, based on predicateB
 - ProB for Siemens in 2008/2009 within Deploy EU project



Aspects of Data Validation

- Focus on expressivity: B language extended (IF-THEN-ELSE, LET for expressions, external functions for string manipulation, regular expressions)
- Tool certification: Tool certified for T2 usage according to EN50128
 - extensive testing and validation and/or double chain
- Full automation, scale to large data values, provide user feedback

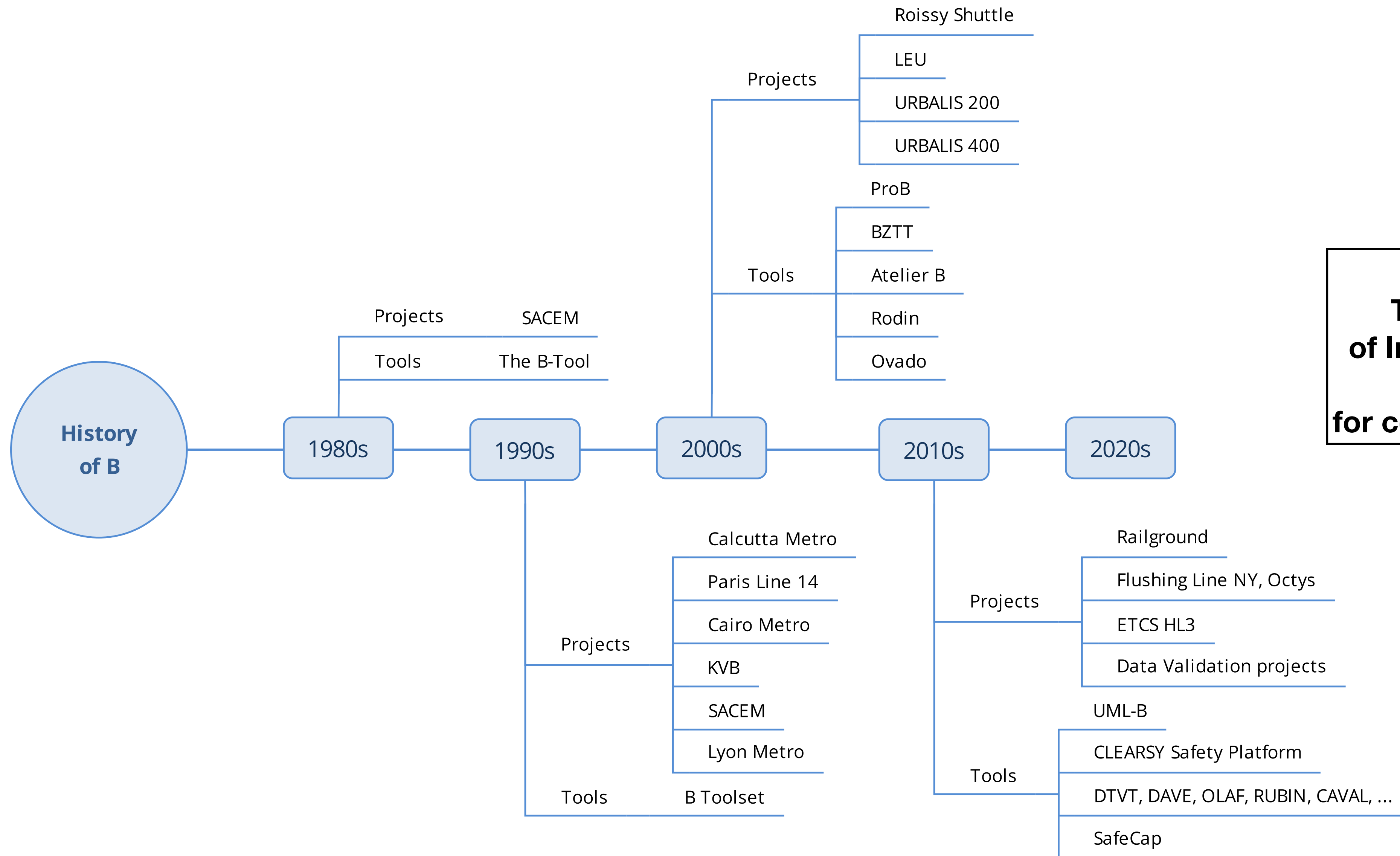


B for **Data Validation: Industrial Uses**

- Line 1 Paris, the second CDGVAL line LISA at the CDG airport in Paris, São Paulo line 4, ALGER line 1, Barcelona line 9, all by Siemens using **RDV** built-on top of **ProB**,
- more metro lines in Paris managed by RATP using **OVADO** which includes a tool developed called predicateB as first chain (development funded by CLEARSY and been maintained and evolved by Systrel for the last 15 years) and **ProB** as secondary tool chain
- Alstom for their URBALIS 400 CBTC system in 2014 using a tool based on **ProB** called **DTVT** developed by CLEARSY for various lines, e.g., in Mexico, Toronto, São Paulo and Panama
- Alstom and SNCF also applied data validation for ETCS-Level 1 software in 2018 using another tool developed by CLEARSY using **ProB**.
- Together with Systrel, Alstom conducted data validation of the Octys CBTC for RATP in 2017 using the **OVADO** tool.
- by Thales using a tool based on ProB called **Rubin** for checking engineering rules of their ETCS Radio Block Centre
- Other tools based on **ProB** were developed by CLEARSY such as **Dave** for General Electric or the latest generation tool called **Caval**.



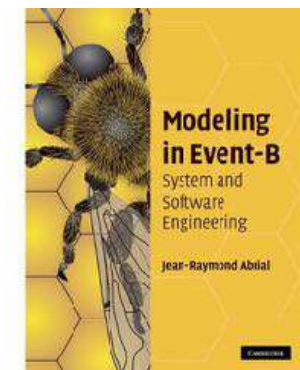
Reflections



**see FMICS'2020 article
The First Twenty-Five Years
of Industrial Use of the B-Method
for common success, fail factors,...**

Summary: B and its Uses

- B for **software development** (classical B): refine specification until B0, apply code generators
 - Line 14 Paris, Alstom U400, ...
 - FM success story, new potential for hardware (LCHIP)
- B for **system modelling** (Event-B): verify critical properties, understand why a system is correct
 - CBTC Flushing Line, NYCT I2S, Octys, Hybrid Level 3, ...
 - Activities have increased in last years, potential for executable models
- B for **data validation**: express properties and B and check data (possibly using a double chain)
 - DTVT, Ovado, Dave, Olaf, Caval, Rubin, ... for Line 1 Paris, Amsterdam, ...
 - FM success story, widespread usage in railway industry



Part II : Commandments and Lessons



for a) using B and



for b) building tools for B, Z, and other formal methods

Inspiration

- Jonathan P. Bowen, Michael G. Hinchey:
Ten Commandments Ten Years On: Lessons for ASM, B, Z and VSR-net. Rigorous Methods for Software Construction and Analysis 2009: 219-233
- Jonathan P. Bowen, Mike G. Hinchey:
Ten Commandments of Formal Methods... Ten Years On. Conquering Complexity 2012: 237-251
- Jonathan P. Bowen, Michael G. Hinchey:
Ten Commandments of Formal Methods. Computer 28(4): 56-63 (1995)
- Jonathan P. Bowen, Michael G. Hinchey:
Seven More Myths of Formal Methods. IEEE Softw. 12(4): 34-41 (1995)
- Jonathan P. Bowen, Michael G. Hinchey:
Seven More Myths of Formal Methods. FME 1994: 105-117
- Anthony Hall:
Seven Myths of Formal Methods. IEEE Softw. 7(5): 11-19 (1990)

Thou Shalt Animate your Models

- ensures your assumptions are consistent (there is at least one model)
- allows to spot errors which are impossible to avoid using invariants and difficult to describe using temporal logic
- spots class of errors you haven't thought of yet

*“Every formal model (proven or not)
which has not been animated contained
errors”*

Christophe Metayer, Systere

(liberal translation from French based on verbal communication)

▼ **M** earley_0

- ✓^R inv2/WD
- ✓^R INITIALISATION/inv1/INV
- ✓^R INITIALISATION/inv2/INV
- ✓^R selector/grd2/WD
- ✓ selector/grd4/WD
- ✓^R selector/inv1/INV
- ✓ selector/inv2/INV
- ✓^R predictor/grd2/WD
- ✓ predictor/grd3/WD
- ✓^R predictor/inv1/INV
- ✓^R predictor/inv2/INV
- ✓^R completer/grd2/WD
- ✓^R completer/grd4/WD
- ✓ completer/grd5/WD
- ✓^R completer/inv1/INV
- ✓ completer/inv2/INV
- ✓^R final/grd1/WD
- ✓ final/grd1/GRD

▼ **M** earley_1

- ✓^R inv1/WD
- ✓^R INITIALISATION/inv2/INV
- ✓ INITIALISATION/inv1/INV
- ✓^R INITIALISATION/act2/SIM
- ✓^R selector/grd1/WD
- ✓ selector/grd2/WD
- ✓ selector/grd4/WD
- ✓^R selector/inv2/INV
- ✓ selector/inv1/INV
- ✓ selector/grd1/GRD
- ✓ selector/act1/WD
- ✓^R predictor/grd1/WD
- ✓ predictor/grd2/WD
- ✓ predictor/grd3/WD
- ✓^R predictor/inv2/INV
- ✓ predictor/inv1/INV
- ✓ predictor/grd1/GRD
- ✓^R predictor/act1/WD
- ✓^R completer/grd1/WD
- ✓ completer/grd2/WD
- ✓^R completer/grd3/WD
- ✓^R completer/grd4/WD
- ✓ completer/grd5/WD
- ✓ completer/inv2/INV
- ✓ completer/inv1/INV
- ✓ completer/grd1/GRD
- ✓ completer/grd3/GRD
- ✓^R completer/act1/WD
- ✓ final/grd1/WD
- ✓ final/grd1/GRD

▼ **M** earley_2

- ✓ inv3/WD
- ✓ inv4/WD
- ✓^R INITIALISATION/inv1/INV
- ✓^R INITIALISATION/inv2/INV
- ✓ INITIALISATION/inv3/INV
- ✓ INITIALISATION/inv4/INV
- ✓^R selector/grd1/WD
- ✓^R selector/grd2/WD
- ✓ selector/grd4/WD
- ✓ selector/grd6/WD
- ✓ selector/inv1/INV
- ✓ selector/inv2/INV
- ✓ selector/inv3/INV
- ✓ selector/inv4/INV
- ✓ selector/grd5/GRD
- ✓ selector/grd1/GRD
- ✓ selector/act1/WD
- ✓ selector/act2/WD
- ✓ predictor/grd1/WD
- ✓ predictor/grd2/WD
- ✓ predictor/grd3/WD
- ✓ predictor/grd5/WD
- ✓ predictor/inv1/INV
- ✓ predictor/inv2/INV
- ✓ predictor/inv3/INV
- ✓ predictor/inv4/INV
- ✓ predictor/grd4/GRD
- ✓ predictor/grd1/GRD
- ✓ predictor/act1/WD
- ✓ predictor/act2/WD
- ✓ completer/grd1/WD
- ✓ completer/grd2/WD
- ✓ completer/grd3/WD
- ✓ completer/grd4/WD
- ✓ completer/grd5/WD
- ✓^R completer/grd7/WD
- ✓ completer/inv1/INV
- ✓ completer/inv2/INV
- ✓ completer/inv3/INV
- ✓ completer/inv4/INV
- ✓ completer/grd6/GRD

▼ **M** earley_3

- ✓^R inv2/WD
- ✓^R thm1/THM
- ✓^R thm2/THM
- ✓^R INITIALISATION/inv1/INV
- ✓^R INITIALISATION/inv2/INV
- ✓^R selector/grd5/WD
- ✓ selector/grd1/WD
- ✓ selector/grd2/WD
- ✓ selector/grd4/WD
- ✓ selector/grd6/WD
- ✓ selector/inv2/INV
- ✓^R selector/grd5/GRD
- ✓ selector/grd1/GRD
- ✓^R selector/grd3/GRD
- ✓^R selector/grd4/GRD
- ✓^R selector/grd6/GRD
- ✓ selector/act1/WD
- ✓ selector/act2/WD
- ✓^R selector/act1/SIM
- ✓^R selector/act2/SIM
- ✓^R predictor/grd4/WD
- ✓ predictor/grd1/WD
- ✓^R predictor/grd2/WD
- ✓ predictor/grd3/WD
- ✓^R predictor/grd5/WD
- ✓ predictor/inv2/INV
- ✓^R predictor/grd4/GRD
- ✓ predictor/grd1/GRD
- ✓^R predictor/grd5/GRD
- ✓^R predictor/act1/WD
- ✓^R predictor/act2/WD
- ✓^R predictor/act1/SIM
- ✓^R predictor/act2/SIM
- ✓^R move_predictor/grd1/WD
- ✓ move_predictor/grd2/WD
- ✓ move_predictor/grd3/WD
- ✓ move_predictor/grd5/WD
- ✓ move_predictor/grd4/WD
- ✓ move_predictor/inv2/INV
- ✓^R completer/grd8/WD
- ✓ completer/grd1/WD

Earley Parsing Algorithm Model

**Fully proven,
but ProB found inconsistency
in the axioms (\rightarrow instead of \rightarrow)
during animation**

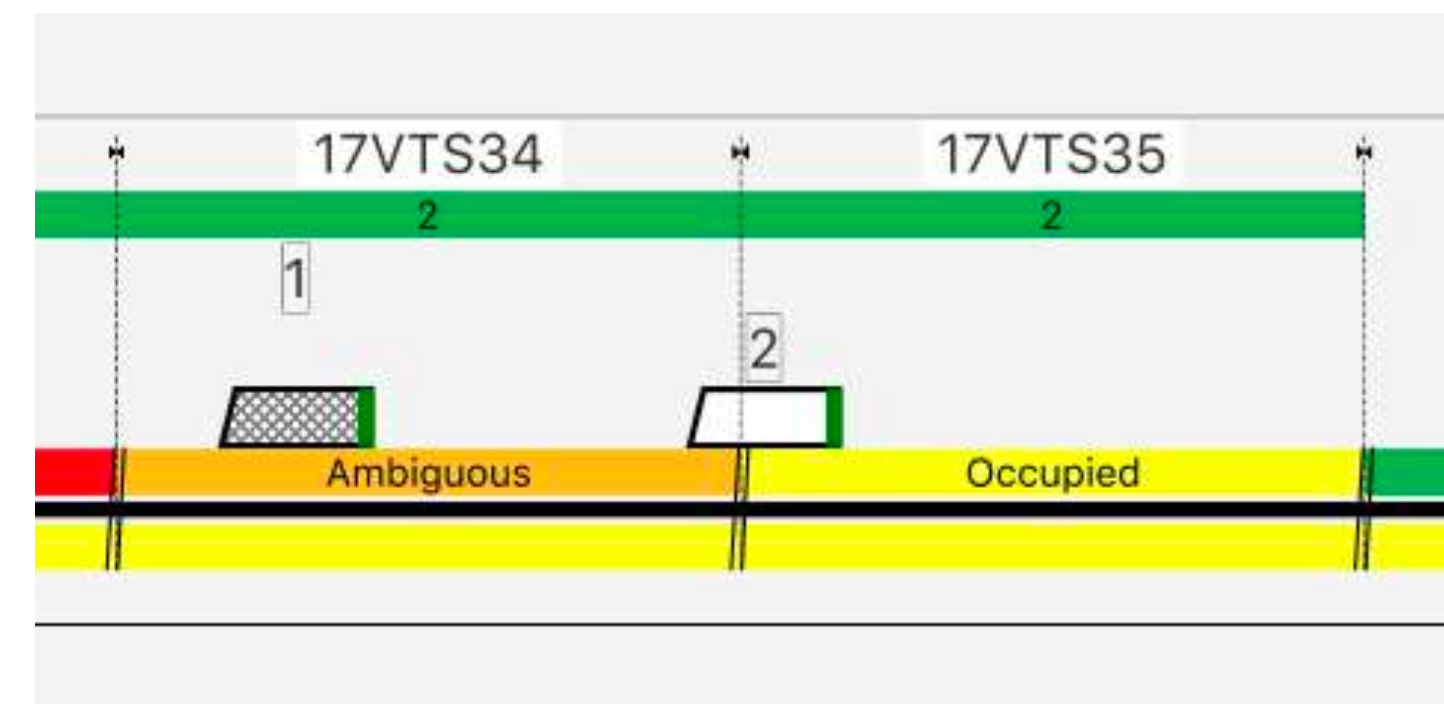
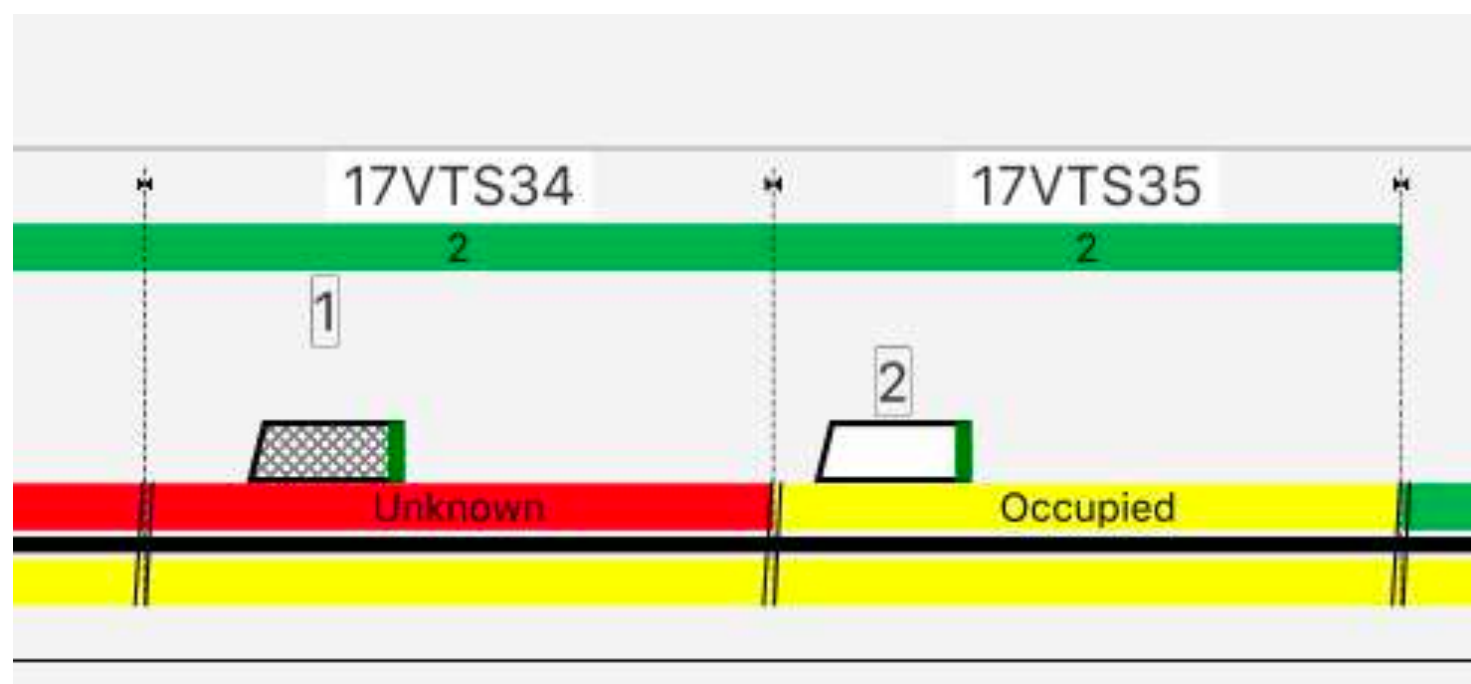


Thou Shalt Visualize 👁️


```

vss_left = {..., "17VTS33" |-> 140, "17VTS34" |-> 240, "17VTS34" |-> 340, ...}
TTD_state = {TTD1|->free, TTD2|->occupied, TTD3|->occupied, ...}
vss_state = {..., "17VTS33" |-> unknown,
                  "17VTS34" |-> unknown, "17VTS35" |-> occupied, ...}
...
Env_train_length = {train1|->30, train2|->30}
  Env_train_FP = {train1|->250, train2|->350}
...
registeredTrains = {train2}
train_reported_integrity = {train2|->confirmed_integrity}

```



ProB2-UI Demo

The screenshot displays the ProB2-UI interface for the 'SimpleTrainTrack.mch' model. The interface is divided into several panes:

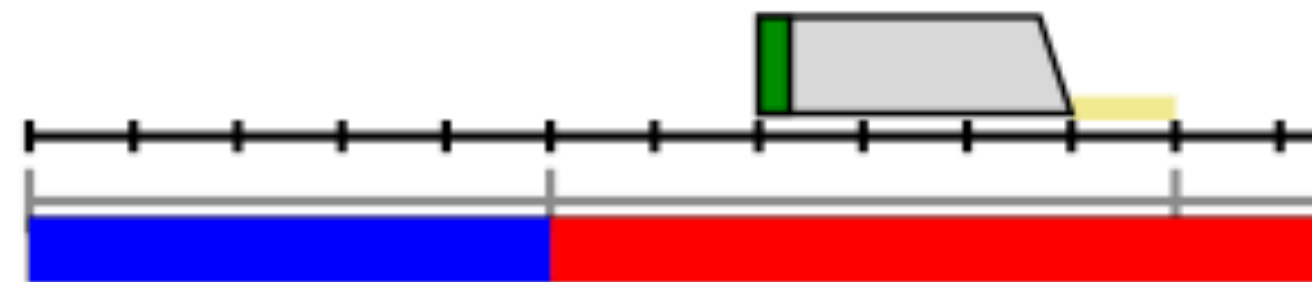
- Operations View:** Located on the top left, it shows a list of operations such as 'TTD_Occupied(ttd=ttd1)', 'TTD_Occupied(ttd=ttd2)', 'TTD_Free(ttd=ttd3)', and 'TrainMoveForward'. It includes a 'Filter Operations' search bar and navigation controls.
- State View:** Located in the top center, it displays the current state of the model. It includes a 'Filter State' search bar and a table of variables and constants. The 'train_rear_end' variable is highlighted with a value of 29. The 'INvariant' section shows several properties that are currently true.
- Project View:** Located on the top right, it shows a list of project components, including 'MovingParticles4', 'WasserkocherEinfach_mch', 'WasserkocherFalsch1_mch', 'WasserkocherFalsch2_mch', 'm0_island_bridge_3cars_mch', and 'm1_bridge_mch'. It also includes a 'History' section showing the sequence of states.
- Replay View:** Located on the bottom left, it shows a 'Status' section with a 'SimpleTrainTrack.prob2trace' file and a 'Replay' button. It also includes a 'Test Case Generation' button.
- Console (REPL):** Located in the bottom center, it shows the 'Interactive Console' with a 'Classical B' prompt and a 'Clear' button. It displays the output of the 'train_rear_end' command.
- VisB View:** Located at the bottom, it shows an 'SVG-based visualization of current state' of the train track, with a blue bar representing the track and a red bar representing the train.

Annotations in the image highlight the following views:

- Operations View** for interactive animation
- State View** to inspect current and preceding state
- Project View** for models and preferences
- Replay View** for automatic trace replay
- Console (REPL)** for interactive exploration
- History View** to inspect and navigating current animation trace
- VisB View** SVG-based visualization of current state

Simple Train Model

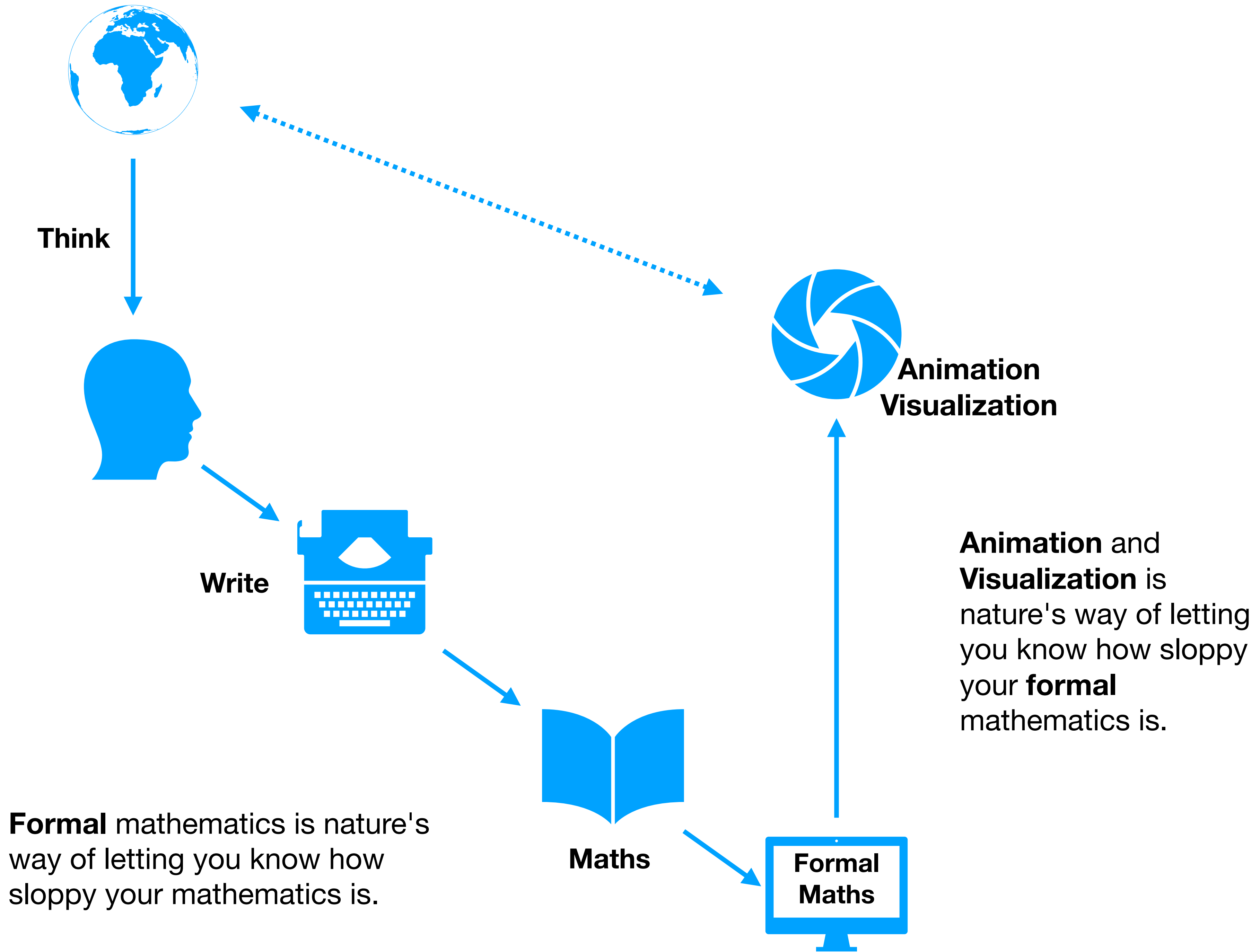
- Track is an interval $0..TrackElementNumber$
- Track is divided into TTD (Trackside Train Detection) zones (e.g. implemented using track circuits or axle counters)
- Trains have positions $train_rear_end(tr)..train_front_end(tr)$
- Some trains have an MA (Movement Authority) extending beyond their front end



- Many things not modelled: delays, position reports, uncertainty of train image, train speed, braking curves, points, train integrity, ...

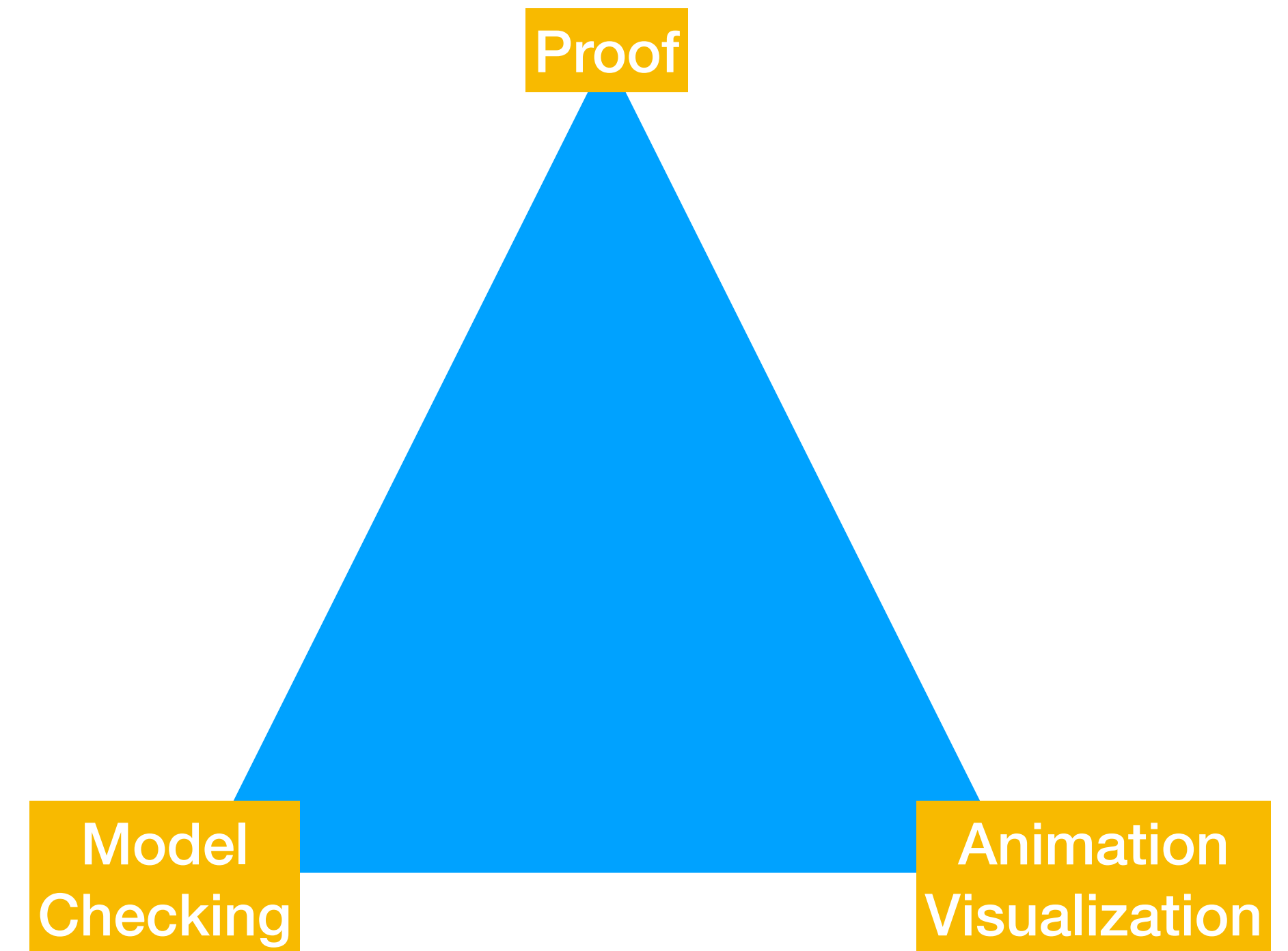
Some Points

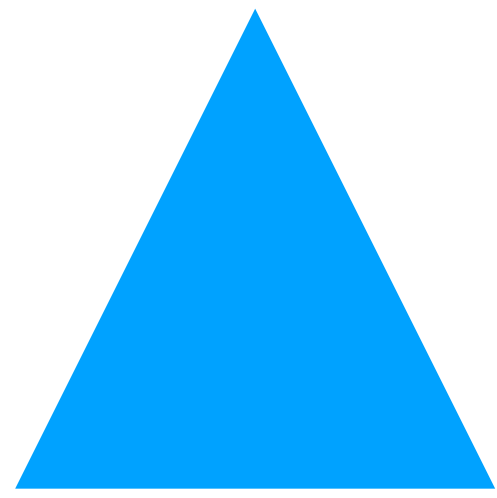
- Animation and Visualisation help me understand models others have written
- also help make your model better understandable to other people, even domain experts not able to read your formal notation



Thou Shalt Not Abandon Thy Traditional Formal Proof Methods

- alternatively:
Thou Shalt Use the Trinity of Methods





The Trinity of Methods

- I tend to use proof, model checking and animation together
- **Proof:** solves the state explosion problem, provides key (inductive) properties and insights
- **Model checking:** finds obvious problems, increases confidence that proof is feasible, checks liveness properties
- **Animation:** validate scenarios (often part of the requirements), find inconsistencies, detect “surprising” and obvious errors quickly

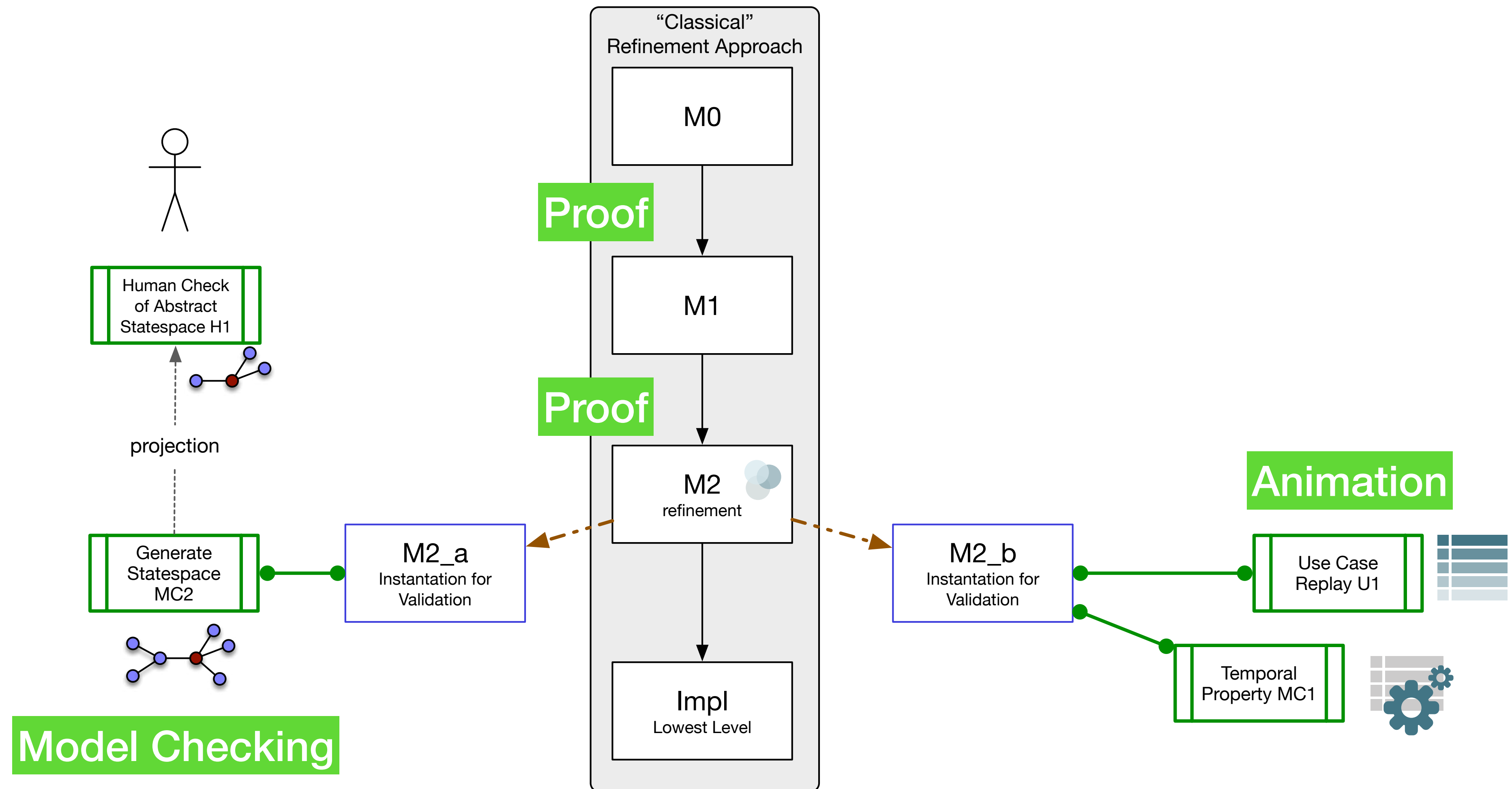
Conflict of Interest

- Proof and animation have conflicting needs:
 - adding an axiomatic property for proof can make finding a valid model much harder

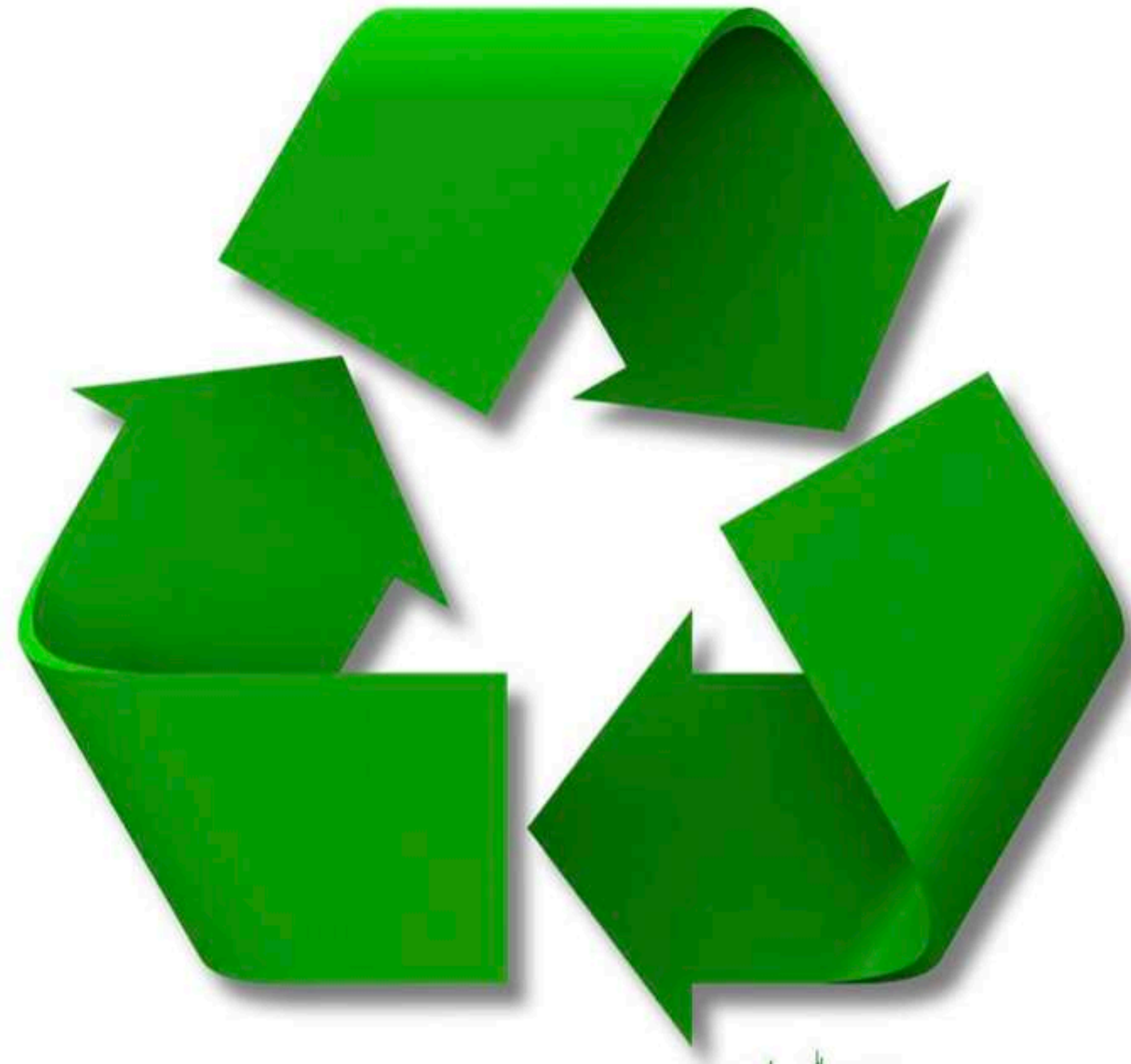
$$\forall s . s \subseteq \text{Track} \Rightarrow P(s)$$

- adding concrete data and constructive definitions for animation can make proof harder and less general
- Use refinement to create model checking and animation instances
- Annotate/isolate complex properties (@prob-ignore pragma)

$$f = \lambda x. x \in \text{Train} \mid \text{front}(x) .. \text{train_ma}(x)$$



Thou Shalt Reuse



Thou Shalt Reuse



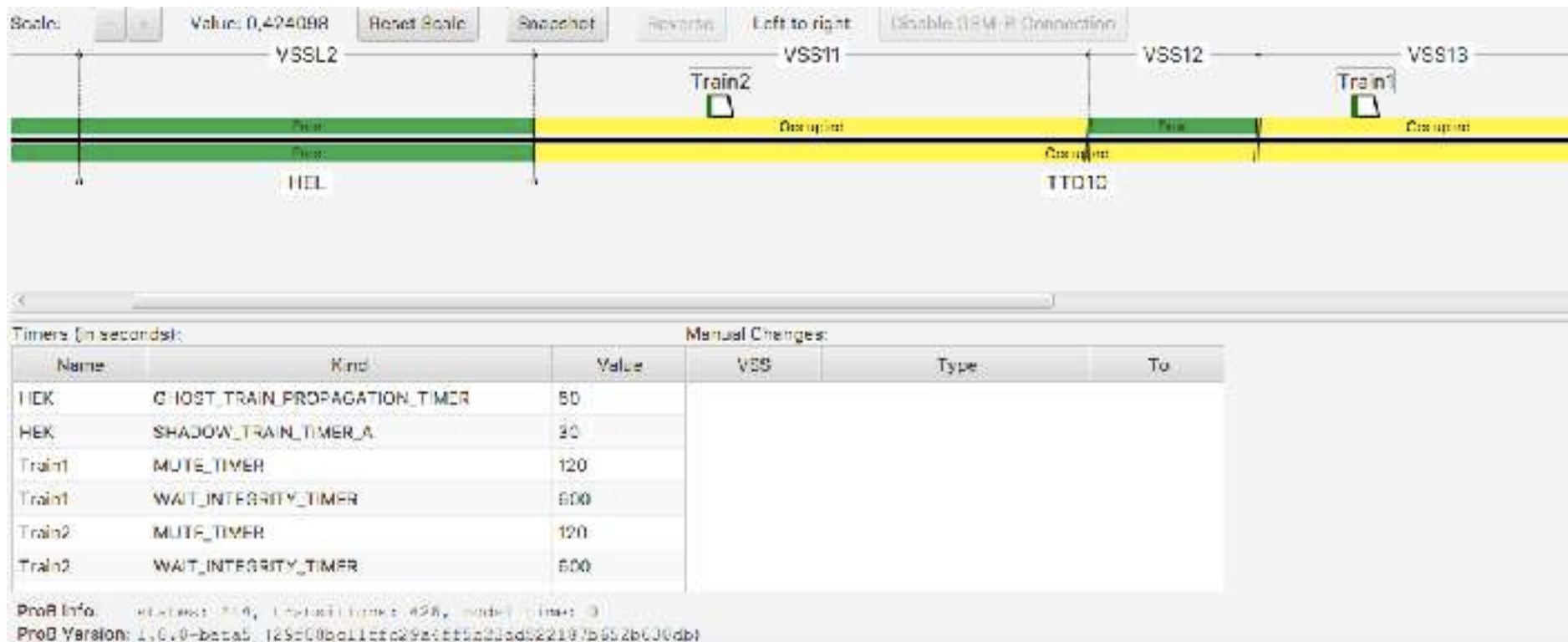
Ideas,
not Models

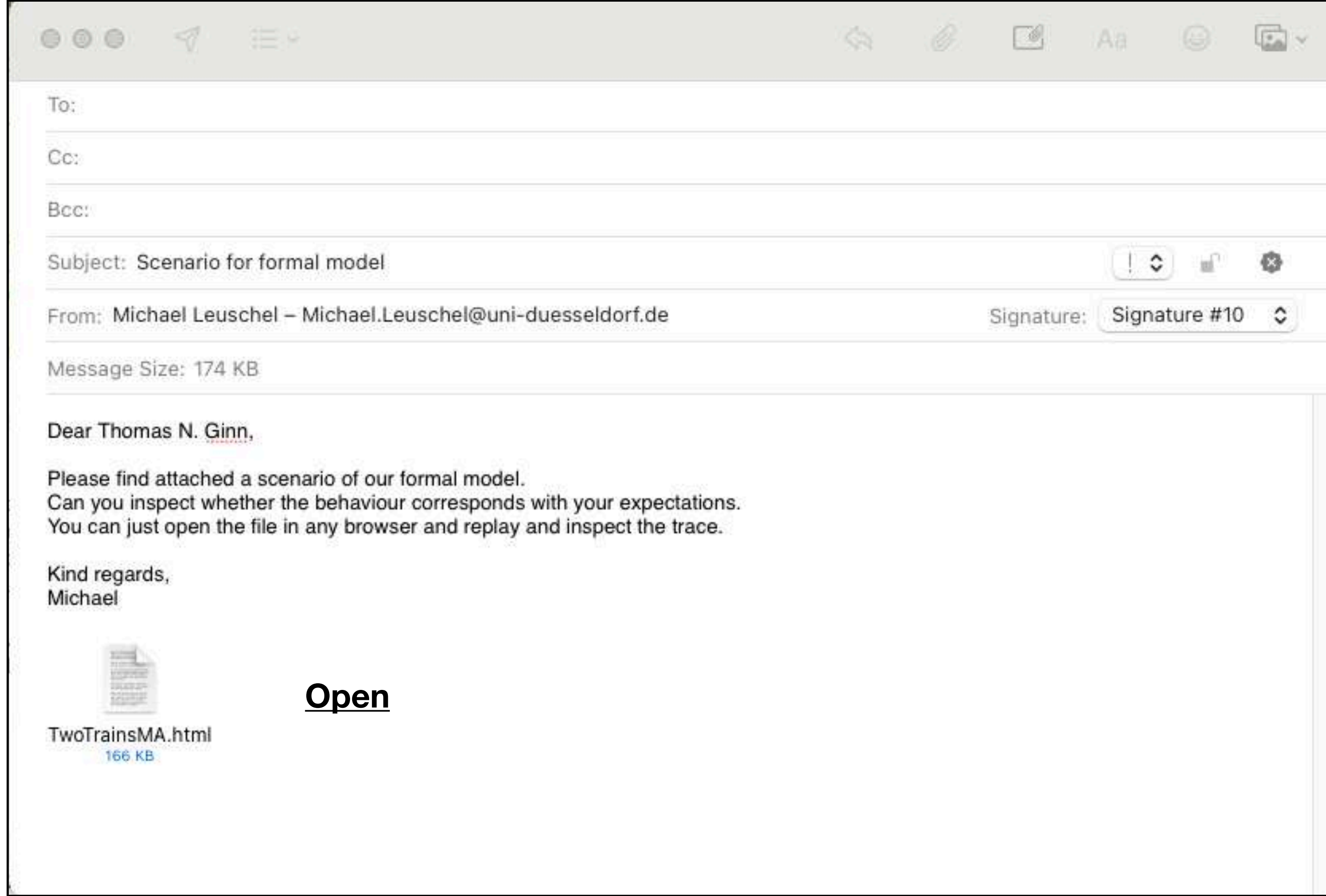
Thou Shalt Reuse Ideas, not Models

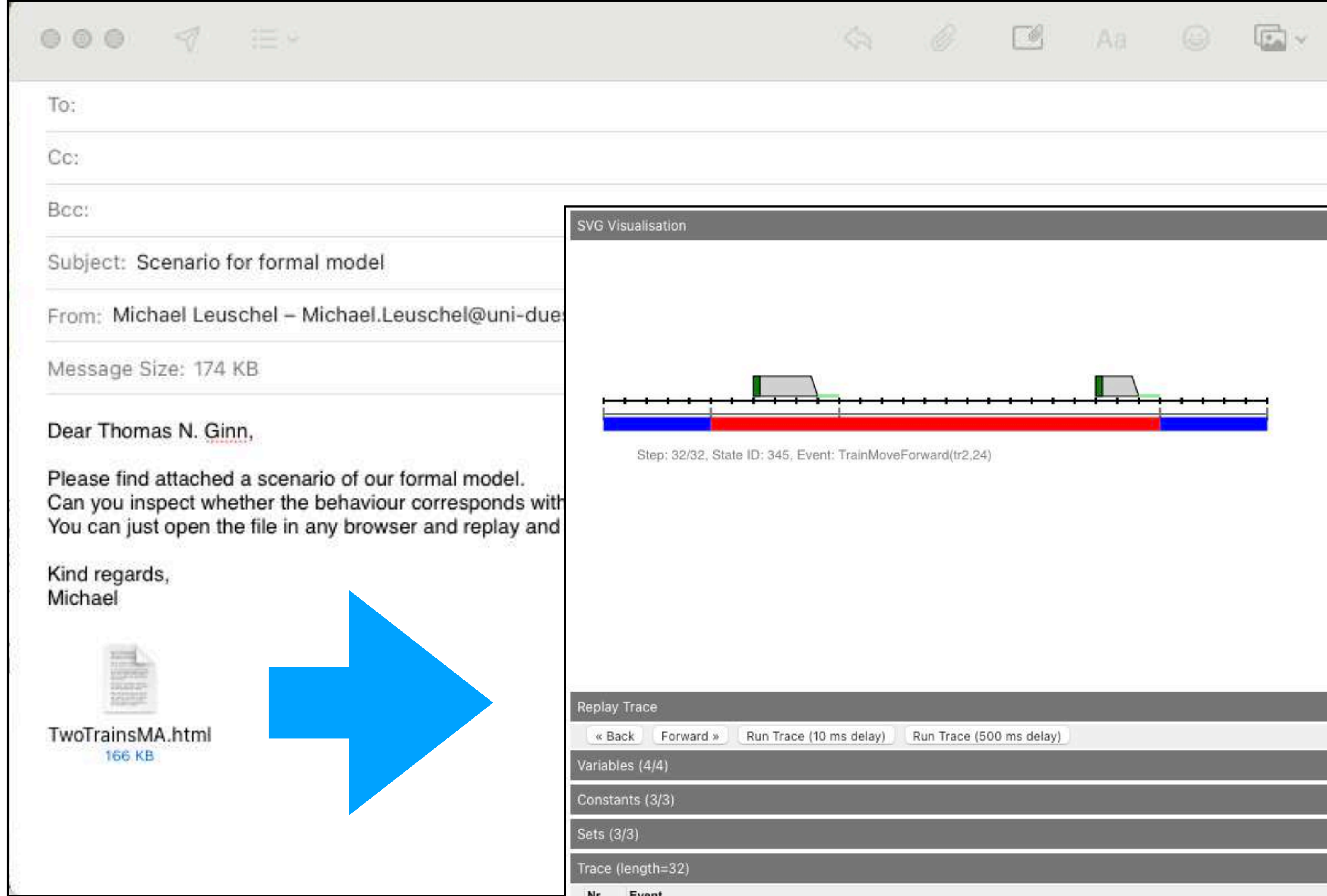
- When modelling:
 - Good idea to reuse: key concepts, ways to decompose a system, approach to ensure inductive proof is possible
 - Usually difficult to reuse formal models for modelling

Thou Shalt Use Models as Documentation


- Static Documentation
- Executable, interactive Documentation (HL3)







SVG Visualisation



Step: 32/32, State ID: 345, Event: TrainMoveForward(tr2,24)

Replay Trace

« Back Forward » Run Trace (10 ms delay) Run Trace (500 ms delay)

Variables (4/4)

Constants (3/3)

Sets (3/3)

Trace (length=32)

Nr	Event
1	SETUP_CONSTANTS(TrackElementNumber=30, TRACK={0,1,2,3,4,5,6,7,8,9,10,11,12,13,14,15,16,17,18,19,20,21,22,23,24,25,26,27,2...
2	INITIALISATION(occ={ttd1,ttd2}, train_rear_end={({tr1 >0}), (tr2 >5)}, train_front_end={({tr1 >2}), (tr2 >6)}, train_ma={})
3	TrainAcceptsFirstMA(tr2,14)
4	TrainMoveForward(tr2,7)

ProB Jupyter Notebooks for Documentation

KISS PASSION Puzzle

A slightly more complicated puzzle (involving multiplication) is the KISS * KISS = PASSION problem.

```
In [3]: 1 {K,P} ⊆ 1..9 ∧  
2 {I,S,A,O,N} ⊆ 0..9 ∧  
3 (1000*K+100*I+10*S+S) *  
4 (1000*K+100*I+10*S+S)  
5 = 1000000*P+100000*A+10000*S+1000*S+100*I+10*O+N ∧  
6 card({K, I, S, P, A, O, N}) = 7
```

Out[3]: *TRUE*

Solution:

- $P = 4$
- $A = 1$
- $S = 3$
- $I = 0$
- $K = 2$
- $N = 9$
- $O = 8$

ProB Jupyter Notebooks for Documentation

Mixing Functional Programming with Constraint Programming

In B we can also define (higher-order) functions and mix those with logical predicates.

```
In [12]: 1 f =  $\lambda x. (x \in \mathbb{Z} \mid x * x) \wedge \text{res} = \{x \mid f(x) = 100\}$ 
```

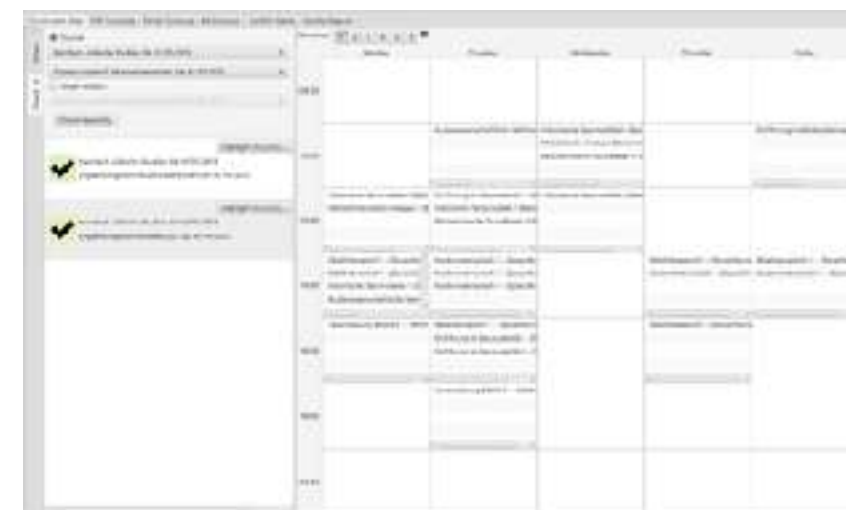
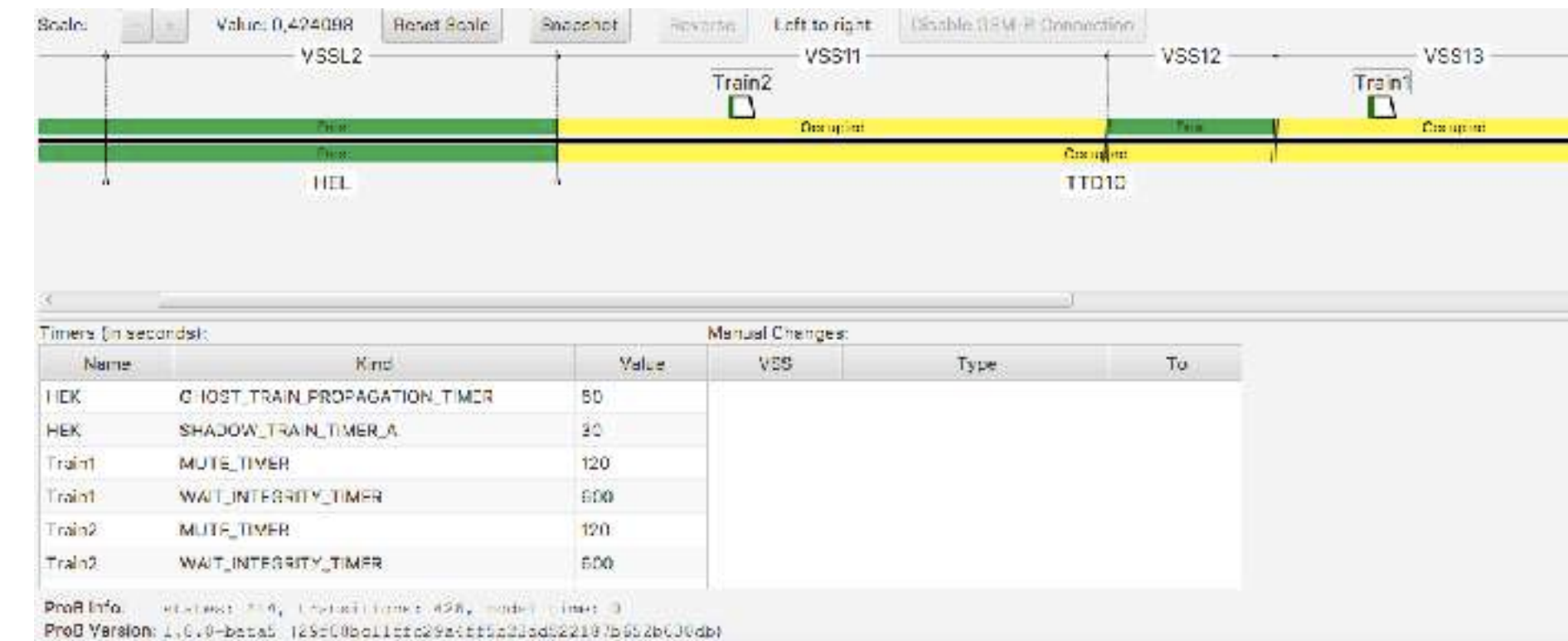
```
Out[12]: TRUE
```

Solution:

- $\text{res} = \{-10, 10\}$
- $f = \lambda x. (x \in \text{INTEGER} \mid x * x)$

Thou Shalt Use Execute your Models

- Oracle in Test Environment (Advance)
- Executable Prototype for early field tests (HL3)
- Long term: maybe formal models in the loop in the final product (Plues tool)



first step (2014):
using B model in real
Alstom test environment

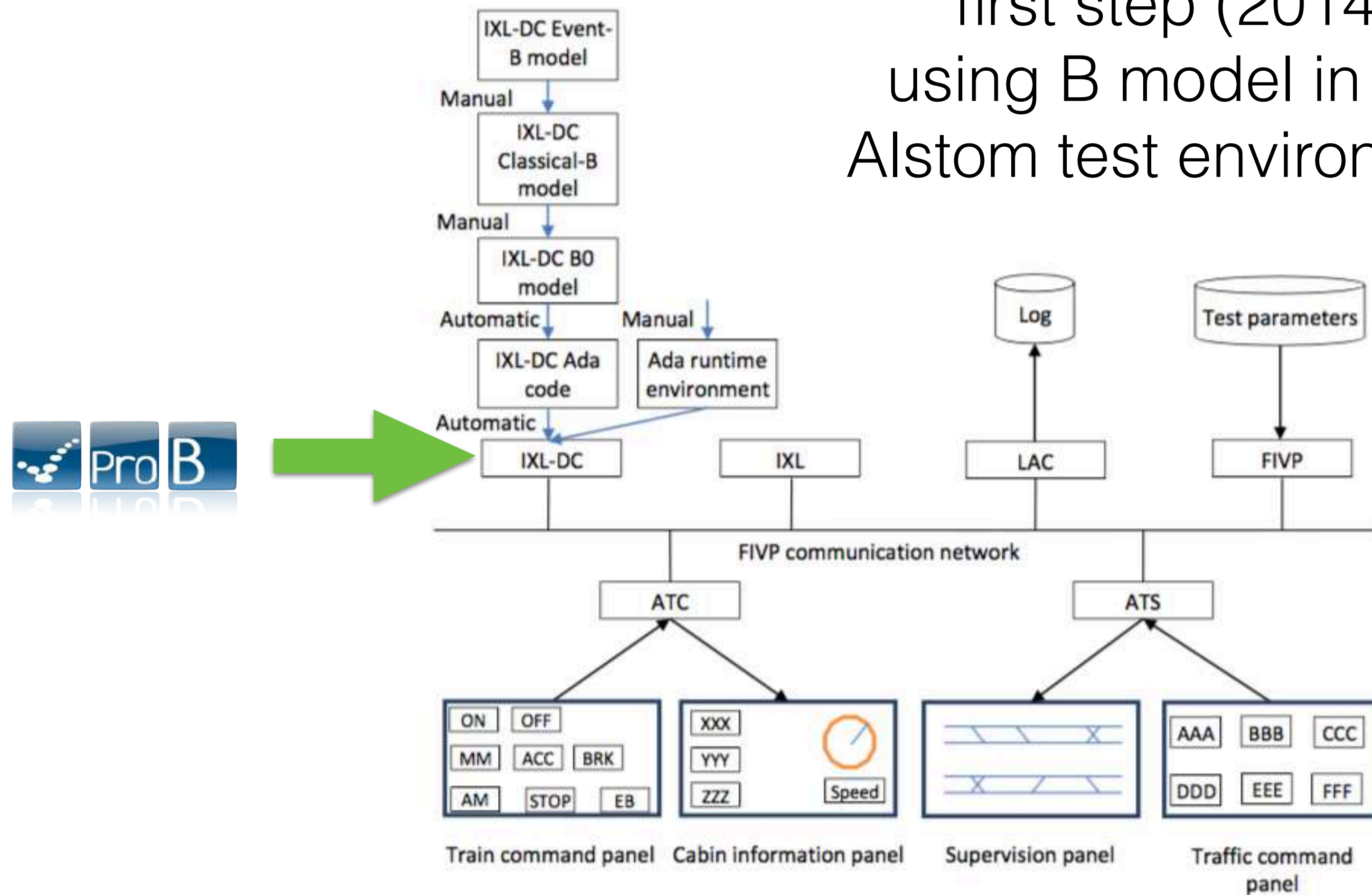


Figure 3.1. Envisioned IXL-DC code generation process and test environment architecture

Project for ProRail

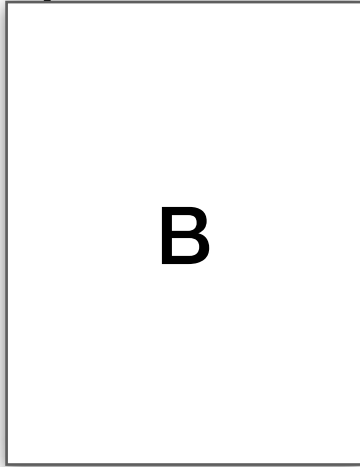
- Field demonstration of the ETCS Hybrid Level 3 (HL3) principles
 - Demonstration by co-operation trackside (Thales, Siemens) and onboard (Alstom, Hitachi)
 - Demonstration line:
ETCS National Integration Facility (ENIF) in Hitchin/UK
- There was insufficient time to model and code a prototype
- But there was sufficient time to embed the formal model at runtime



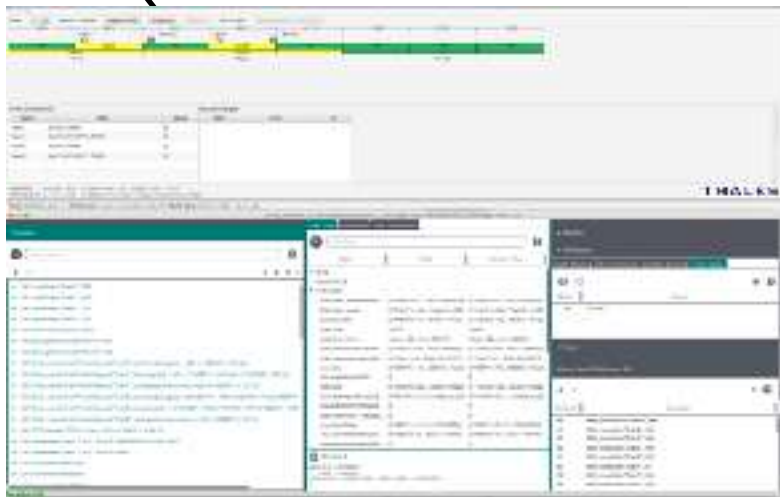


Natural
Language
Specification

New System
New Feature



Formal
Model



Testing,
Debugging



Execution in
real
environment

ProRail Project Summary

- using model as demonstrator/prototype is feasible, there were a lot of technical issues, ProB was not one of them!
- animation/visualization can help understand and debug a given specification
- using a formal model allowed to quickly adapt the model as fixes for issues came along, several new requirements were integrated
- log of formal model could be replayed step-by-step to analyse issues



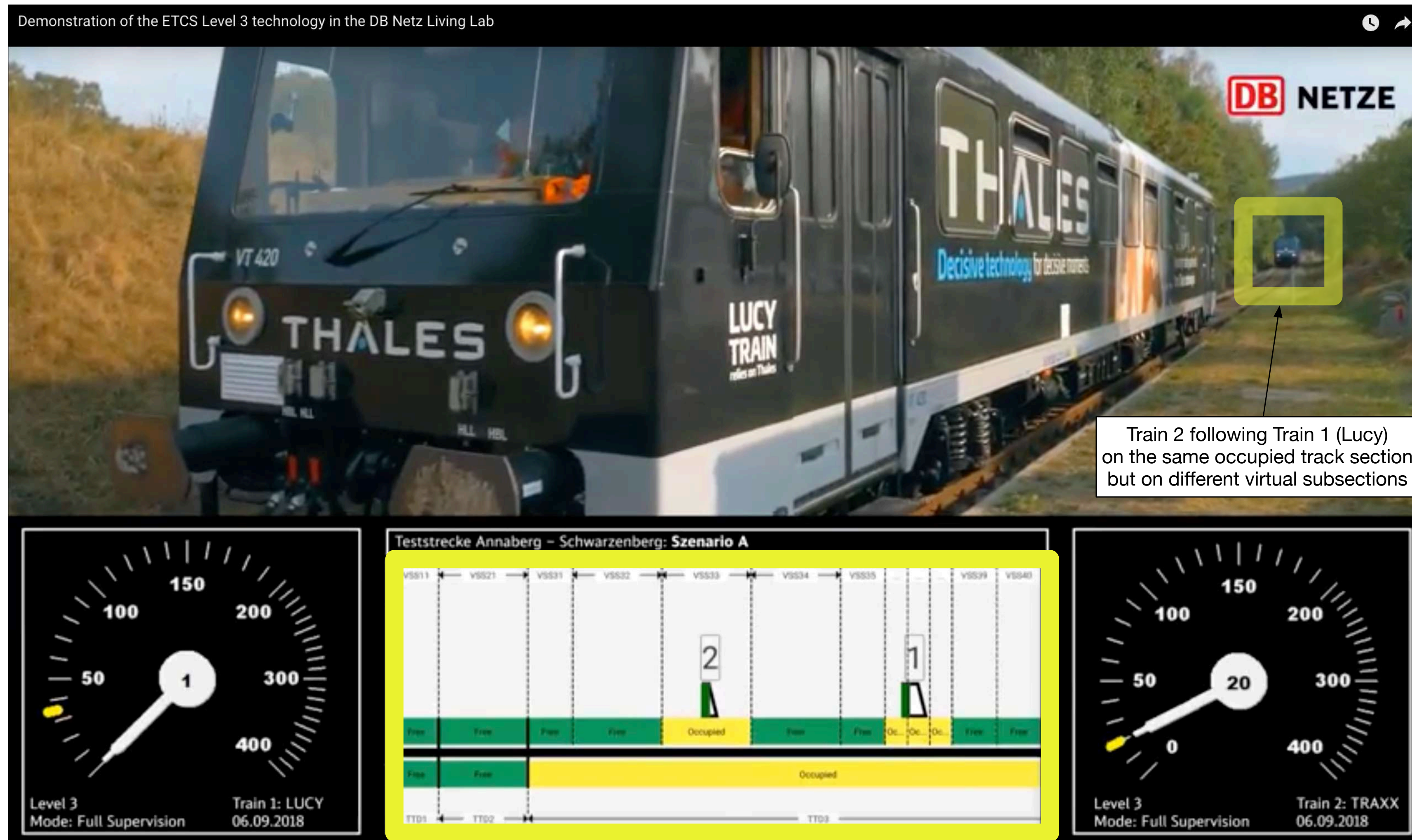
The use of formal methods in specification and demonstration of ERTMS Hybrid Level 3

Prepared on behalf of the International Technical Committee
by Maarten Bartholomeus, Bas Luttik, Tim Willemse,
Dominik Hansen, Michael Leuschel and Paul Hendriks

IRSE News
November 2019



ProB in Action : Formal Models in Realtime



ProB running in real-time animating a
formal B model of the **Hybrid-Level 3** principles
developed by a team from the University of Düsseldorf and Thales with support from ClearSy

**from modelling
to tools**



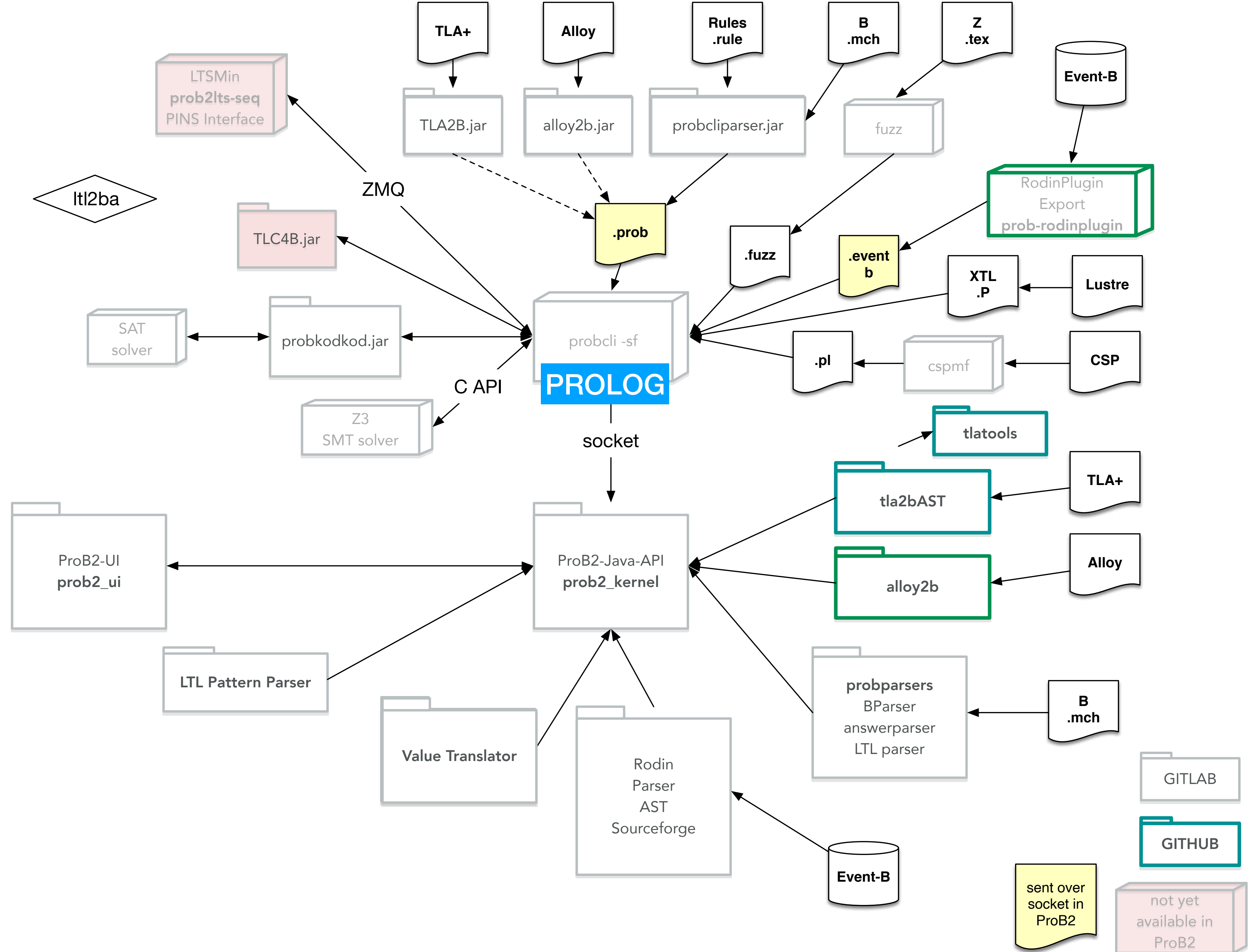
Thou Shalt Choose an Appropriate Notation/ Programming Language for your Tool

- Prolog for type checking, rule-based theorem proving, constraint solving
- Java, Tcl/Tk, ... for user interface
- C for LTL model checking
- ...

The heart of ProB is written in Prolog

but other languages are used around it:

Java
C, C++
Tcl/Tk
Haskell



Hindley-Milner Type Inference

- Easy to encode in Prolog: one type inference rule is one Prolog clause
- More powerful than Atelier-B, ...
- cf. VPT-2020 article
- Fast

```
type([],set(_)) --> !, [].
type(union(A,B),set(R)) --> !,type(A,set(R)), type(B,set(R)).
type(intersect(A,B),set(R)) --> !,type(A,set(R)), type(B,set(R)).
type(plus(A,B),integer) --> !,type(A,integer), type(B,integer).
type(in_set(A,B),predicate) --> !,type(A,TA), type(B,set(TA)).
type(gt(A,B),predicate) --> !,type(A,integer), type(B,integer).
type(and(A,B),predicate) --> !,type(A,predicate),type(B,predicate).
type(eq(A,B),predicate) --> !,type(A,TA),type(B,TA).
type(Nr,integer) --> {number(Nr)},!.
type([H|T],set(TH)) --> !,type(H,TH), type(T,set(TH)).
type(ID,TID) --> {identifier(ID)},\+ defined(id(ID,_)),!,
                    add((id(ID,TID))). % creates fresh variable
type(ID,TID) --> {identifier(ID)},defined(id(ID,TID)),!.
type(Expr,T,Env,_) :-
    format('Type error for ~w (expected: ~w, Env: ~w)~n',[Expr,T,Env]),fail.

defined(X,Env,Env) :- member(X,Env).
add(X,Env,[X|Env]).

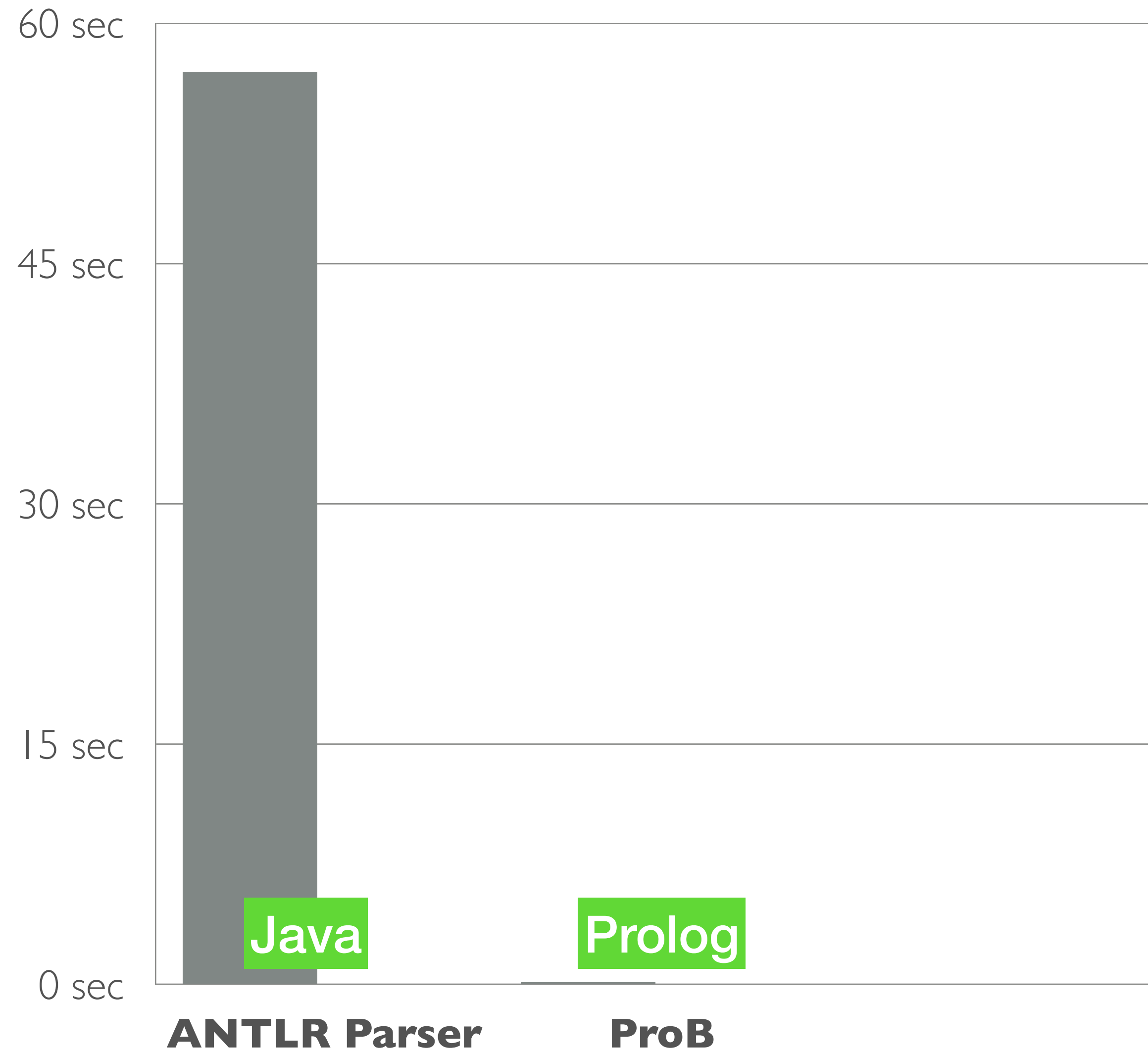
identifier(ID) :- atom(ID), ID \= [].

type(Expr,Result) :- type(Expr,Result,[],Env), format('Typing env: ~w~n',[Env]).
```

$$\{z\} \cup \{x,y\} = u \wedge z > v$$

```
| ?- type(and(eq(union([z],[x,y]),u),gt(z,v)),R).
Typing env: [id(v,integer),id(u,set(integer)),id(y,integer),id(x,integer),id(z,integer)]
R = predicate ?
yes
```

Anecdotal Evidence: Typechecking 8000 Line B specification



Semantic Translation Rules: Alloy2B

- Translator of Alloy [Jackson] to B
 - Adaptation of formal semantics of Alloy by simply using B syntax
- Rules can be translated to Prolog clauses
- First version was written in Kotlin (JVM), then switched to Prolog as error prone and tedious to encode rules

$$E[p + q]i \hat{=} E[p]i \cup E[q]i$$

$$E[p \& q]i \hat{=} E[p]i \cap E[q]i$$

$$E[p - q]i \hat{=} E[p]i \setminus E[q]i$$

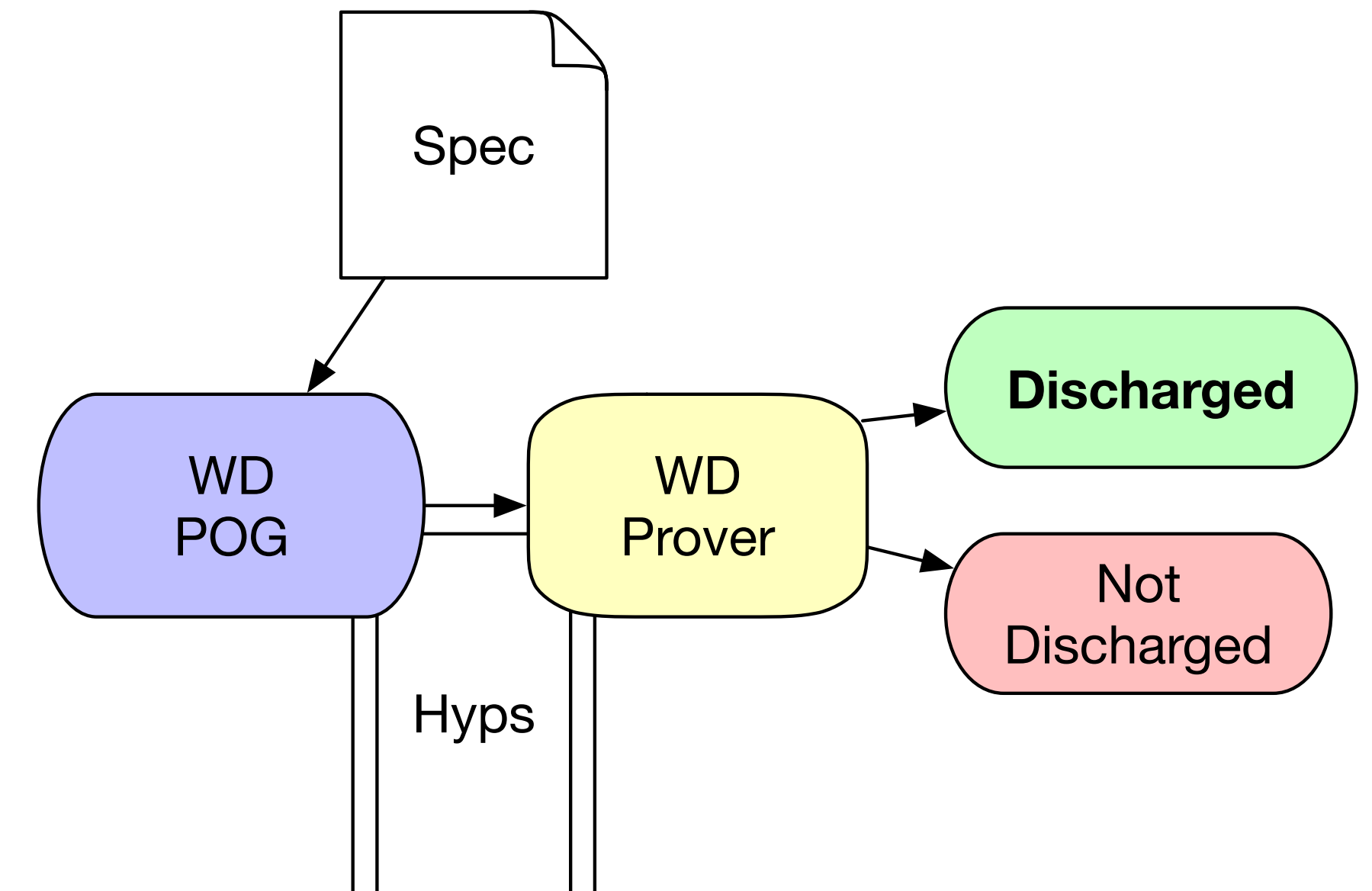
```
translate_binary_e_p(Binary, TBinary) :-  
    Binary =.. [Op,P,Q,_,POS],  
    alloy_to_b_binary_operator(Op, BOp),  
    translate_e_p(P, TP),  
    translate_e_p(Q, TQ),  
    translate_pos(POS, BPOS),  
    TBinary =.. [BOp,BPOS,TP,TQ].
```

```
alloy_to_b_binary_operator(plus, union).  
alloy_to_b_binary_operator(intersection, intersection).  
alloy_to_b_binary_operator(minus, set_subtraction).  
alloy_to_b_binary_operator(implication, implication).  
alloy_to_b_binary_operator(iff, equivalence).
```

...

Prolog Theorem Prover for Proving Well-Definedness

- WD Prover [iFM'2020] to prove absence of division by zero, undefined function applications, cardinality of infinite sets, ...
- Shared Hypothesis stack
 - pop via **Prolog backtracking**
 - Only **logarithmic** accesses to Hypotheses
 - Efficient **rule-based prover** using **Prolog unification**



Patterns for Lookups	
$finite(A)$	$A \in B$
$A = B$	$A \neq B$
$A \leq B$	$A \geq B$
$A \subseteq B$	$A \supseteq B$



One WD Prover Rule



“The range of a function is finite if the function is finite.”

```
check_finite(range(A), Hyp, ran(PT)) :- !,  
                                     check_finite(A, Hyp, PT)
```

Prolog



In Java:

82 lines of code

(9 lines are copyright notice)

The Prolog code is also very flexible:

it can be used for finding proofs

but also for re-playing or

checking proofs if the proof tree argument is provided

```

/*****
 * Copyright (c) 2007, 2014 ETH Zurich and others.
 * All rights reserved. This program and the accompanying materials
 * are made available under the terms of the Eclipse Public License v1.0
 * which accompanies this distribution, and is available at
 * http://www.eclipse.org/legal/epl-v10.html
 *
 * Contributors:
 *   ETH Zurich - initial API and implementation
 *****/
package org.eventb.internal.core.seqprover.eventbExtensions;

import org.eventb.core.ast.Expression;
import org.eventb.core.ast.FormulaFactory;
import org.eventb.core.ast.Predicate;
import org.eventb.core.ast.SimplePredicate;
import org.eventb.core.ast.UnaryExpression;
import org.eventb.core.seqprover.IProofMonitor;
import org.eventb.core.seqprover.IProverSequent;
import org.eventb.core.seqprover.IReasonerInput;
import org.eventb.core.seqprover.IReasonerOutput;
import org.eventb.core.seqprover.ProverFactory;
import org.eventb.core.seqprover.ProverRule;
import org.eventb.core.seqprover.SequentProver;
import org.eventb.core.seqprover.IProofRule.IAntecedent;
import org.eventb.core.seqprover.eventbExtensions.Lib;
import org.eventb.core.seqprover.reasonerInputs.EmptyInputReasoner;

public class FiniteRan extends EmptyInputReasoner {

    public static final String REASONER_ID = SequentProver.PLUGIN_ID + ".finiteRan";

    @Override
    public String getReasonerID() {
        return REASONER_ID;
    }

    @ProverRule("FIN_REL_RAN_R")
    protected IAntecedent[] getAntecedents(IProverSequent seq) {
        Predicate goal = seq.goal();

        // goal should have the form finite(ran(r))
        if (!Lib.isFinite(goal))
            return null;
        SimplePredicate sPred = (SimplePredicate) goal;
        if (!Lib.isRan(sPred.getExpression()))
            return null;

        // There will be 1 antecedents
        IAntecedent[] antecedents = new IAntecedent[1];

        UnaryExpression expression = (UnaryExpression) sPred.getExpression();

        Expression r = expression.getChild();

        final FormulaFactory ff = seq.getFormulaFactory();
        // finite(r)
        Predicate newGoal = ff.makeSimplePredicate(Predicate.KFINITE, r, null);
        antecedents[0] = ProverFactory.makeAntecedent(newGoal);

        return antecedents;
    }

    protected String getDisplayName() {
        return "finite of range of a relation";
    }

    @Override
    public IReasonerOutput apply(IProverSequent seq, IReasonerInput input,
                                IProofMonitor pm) {
        IAntecedent[] antecedents = getAntecedents(seq);
        if (antecedents == null)
            return ProverFactory.reasonerFailure(this, input,
                                                "Inference " + getReasonerID()
                                                + " is not applicable");

        // Generate the successful reasoner output
        return ProverFactory.makeProofRule(this, input, seq.goal(),
                                            getDisplayName(), antecedents);
    }
}

```

Java

WD Prover

```
check_finite(range(A), Hyp, ran(PT)) :- !,  
    check_finite(A, Hyp, PT)
```

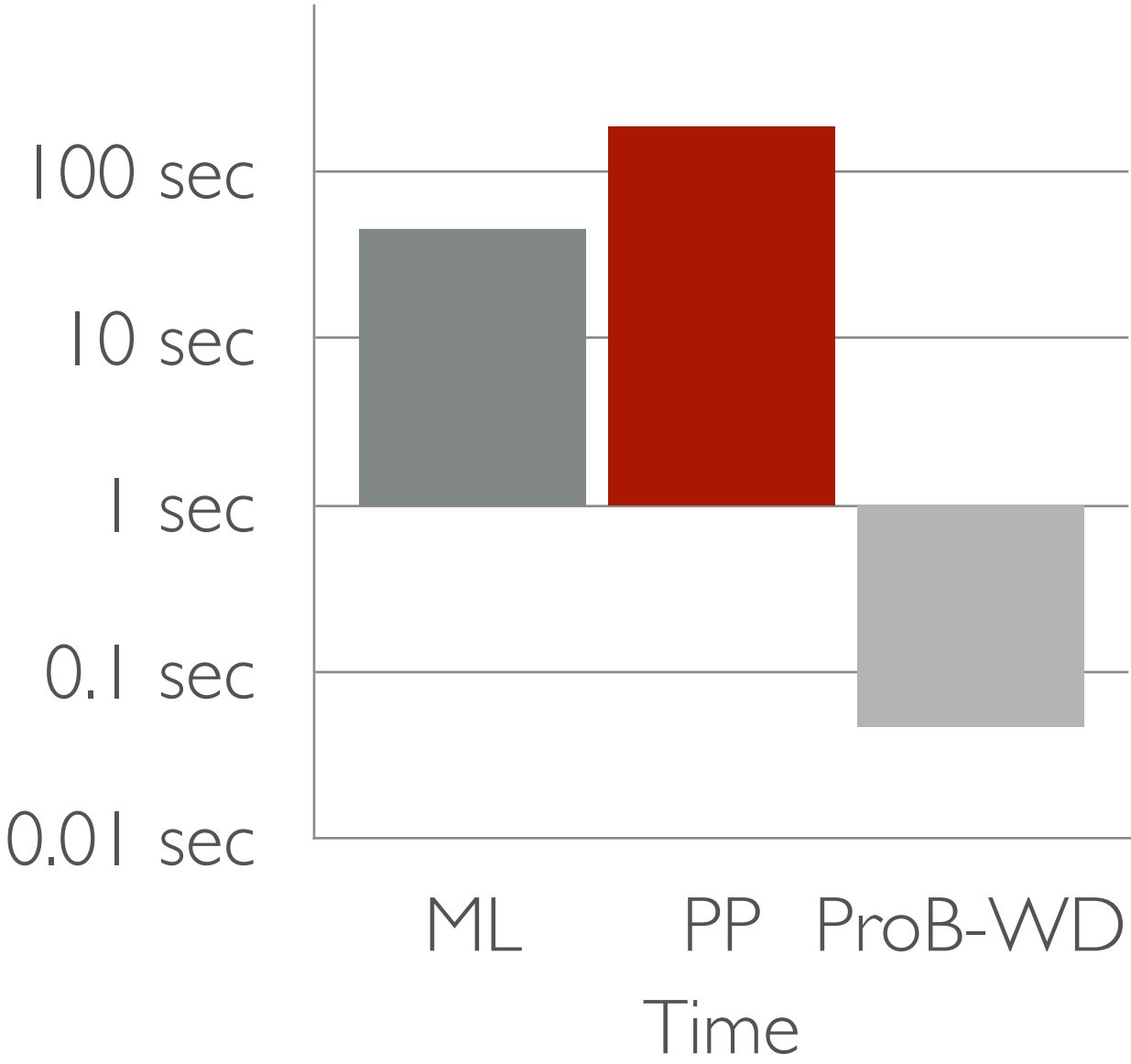
Prolog

Java

```
public class FiniteRan extends EmptyInputReasoner {  
  
    public static final String REASONER_ID = SequentProver.PLUGIN_  
  
    @Override  
    public String getReasonerID() {  
        return REASONER_ID;  
    }  
  
    @ProverRule("FIN_REL_RAN_R")  
    protected IAntecedent[] getAntecedents(IProverSequent seq) {  
        Predicate goal = seq.goal();  
  
        // goal should have the form finite(ran(r))  
        if (!Lib.isFinite(goal))  
            return null;  
        SimplePredicate sPred = (SimplePredicate) goal;  
        if (!Lib.isRan(sPred.getExpression()))  
            return null;  
  
        // There will be 1 antecedents  
        IAntecedent[] antecedents = new IAntecedent[1];  
  
        UnaryExpression expression = (UnaryExpression) sPred.getEx  
  
        Expression r = expression.getChild();  
  
        final FormulaFactory ff = seq.getFormulaFactory();  
        // finite(r)  
        Predicate newGoal = ff.makeSimplePredicate(Predicate.KFINI  
        antecedents[0] = ProverFactory.makeAntecedent(newGoal);  
    }  
}
```

Biased Benchmarks

- Test Atelier-B provers ML, PP and Z3 on 413 POs from ProB regression tests
- **Biased**, but shows that our prover is **fast** and can prove POs not proven by existing provers (in default settings)



Prover	Proved	Unproved	Ctrl-C	Min.	Max.	Total	w/o Min.
ProB-WD	413	0	0	0.000 sec	0.006 sec	0.047 sec	0.047 sec
ML	190	223	0	0.260 sec	18.725 sec	152.633 sec	45.253 sec
PP	230	165	18	0.092 sec	49.144 sec	222.940 sec	186.600 sec
						1017.406 sec	-
Z3	9	14	0	0.007 sec	2.520 sec	crash	-

A few more commandments

- Thou Shalt Honour Text and Command-Line Interfaces:
 - Text and command-line tools are not dead yet
- Thou Shalt Think Twice before Redeveloping your Tool from Scratch
 - Redeveloping a tool or even a UI from scratch is hard and will take longer than you think
- Thou Shalt Obtain User Feedback and User Your Own Tool
 - make it easy for users to provide feedback; use your tool yourself



Summary: Thou Shalt

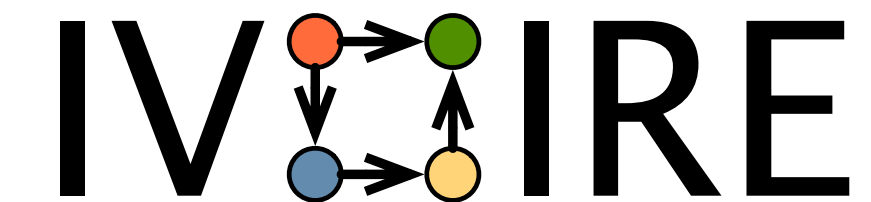
1. Animate Your Models
2. Visualize Your Models
3. Reuse Ideas, not Models
4. Not Abandon Thy Traditional Formal Proof Methods
5. Use Models as Documentation
6. Execute Your Models
7. Choose an Appropriate Notation/Programming Language for Your Tool
8. Honour Text and Command-Line Interfaces
9. Think Twice before Redeveloping your Tool from Scratch
10. Obtain User Feedback and Use Your Own Tool

I. Thou Shalt

II. Thou Shalt

Outlook: New Projects

- IVOIRE (with Univ. Linz)
 - Automate Validation in a Refinement-based Development Process
- KI-LOK
 - Certifying Railway Systems with AI



Formal Methods in Industry

- Formal methods in general and **B** in particular can be used to verify/validate:
 - **code** of individual components
 - **systems** consisting of components (algorithms, design,...)
 - **configurations** of components
- Formal methods have become much more useful thanks to **progress** in proof, constraint solving (SAT, SMT, CLP), visualization
 - tools can find errors which no human could ever find
 - tools can guarantee the absence of certain errors
 - tools can help visualize and understand very complex systems, behaviours and interactions

Danny De Schreye
Maurice Bruynooghe
Bart Demoen
Marc Denecker
Bern Martens
Wim Vanhoof

Robert Glück
Neil D. Jones
Jesper Jørgensen
Torben Mogensen

Fabio Fioravanti
Alberto Pettorossi
Maurizio Proietti
Emanuele De Angelis

John Gallagher
Manual Hermenegildo
German Puebla
Josep Silva
Salvador Tamarit
German Vidal

Kostis Sagonas
and many more

Thank you very much



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Nadine Elbeshausen

Fabian Fritz

Marc Fontaine

Marc Frappier

David Gelessus

Stefan Hallerstede

Dominik Hansen

Christoph Heinzen

Michael Jastram

Philipp Körner

Sebastian Krings

Lukas Ladenberger

Li Luo

Thierry Massart

Daniel Plagge



Antonia Pütz

Mireille Samia

Joshua Schmidt

David Schneider

Corinna Spermann

Sebastian Stock

Yumiko Takahashi

Edd Turner

Michelle Werth

Dennis Winter

Fabian Vu

Alstom (Fernando Mejia,...)

ClearSy (Thierry Lecomte,...)

Siemens

Systerel

Thales (Nader Nayeri, Georg Hemzal,...)

