Qtpi: Simulating (Concurrent) Quantum Systems

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To begin

Communicating Quantum Processes (Gay and Nagarajan, POPL 2005) describes a programming language.

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Qtpi is an implementation of modified CQP, with a symbolic quantum calculator.

It's available on github at mdxtoc/qtpi, with lots of examples and documentation. Can do BB84 QKD and other protocols, Grover's algorithm, W state calculation, and more.

What's interesting?

1. ~~Quantum stuff~~
2. Language
3. Symbolic calculator
4. Probabilistic execution
5. Resource accounting for qubits
6. ~~Qbit collections~~
7. ~~Overloaded operators~~
8. ~~Sparse matrix tricks~~
9. ~~Iterative constructs~~
10. Demos

Protocol steps

A protocol agent can:

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- obtain a qubit;

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- put a qubit (or qubits) through a quantum gate;

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Protocol steps

A protocol agent can:

- obtain a qubit;
- put a qubit (or qubits) through a quantum gate;
- measure a qubit;
- send or receive a qubit;
- send or receive a classical value, such as a list of numbers or bits;
- do some classical calculation.

Process notation

Based on Milner's pi calculus. Very stark.

$$\textit{procdef} \quad ::= \quad p(x,\ \ldots, z) = P$$

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Communication channels can be sent and received in messages

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Communication channels can be sent and received in messages; so can qubits.

Qtpi is strongly typed (Hindley-Milner). Explicit typing is optional (but, for simplicity, not described).

Expression notation

There's a functional notation for calculation, which looks like Miranda ...

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To deal with qubit accounting, functions can't have anything to do with qubits.

What's interesting (again)?

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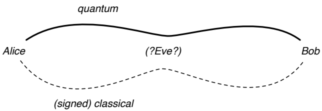
10. Demos

Demo

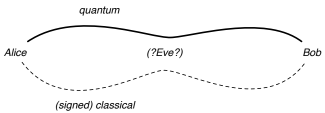
Teleportation, queen of the baby protocols

BB84, queen of the QKD protocols



- generate a one-time code without transmitting it.

BB84, queen of the QKD protocols



- generate a one-time code without transmitting it.
- Alice chooses 1000 bits (say);

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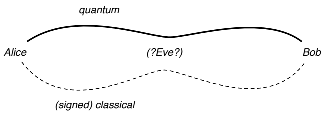

- generate a one-time code without transmitting it.
- Alice chooses 1000 bits (say);
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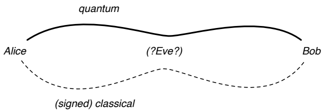

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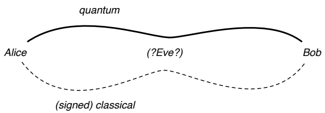

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- They compare notes (classically) about their random choices;

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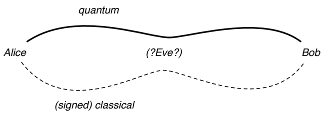

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- They compare notes (classically) about their random choices;
- If Eve has not intervened, they share $\sim$500 secret bits;

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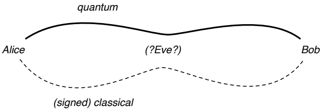

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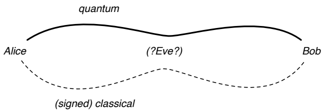

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- They compare notes (classically) about their random choices;
- If Eve has not intervened, they share $\sim$500 secret bits;
- Bob sends Alice (classically) a random sample of n of his bits;
- Only a $(\frac{3}{4})^n$ chance that Eve has meddled and those bits match;

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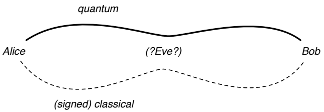

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- Bob sends Alice (classically) a random sample of n of his bits;
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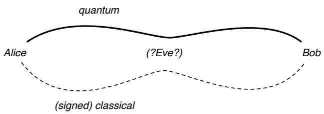

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- If Eve has not intervened, they share $\sim$500 secret bits;
- Bob sends Alice (classically) a random sample of n of his bits;
- Only a $(\frac{3}{4})^n$ chance that Eve has meddled and those bits match;
- Otherwise A & B share a (500-n)-bit secret one-time code;
- Alice uses it to XOR the message and send it classically.