”La main à la pâte”, An Abacus to Teach Formal Specifications

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Abstract

This paper describes the use of a soroban (japanese abacus) for teaching abstraction and refinement in a introductory course on B specifications.

Keywords: B method, abacus, numeral systems, abstraction, refinement

1. INTRODUCTION

In 1995 Nobel Prize winner Georges Charpak launched (after L.M. Lederman in Chicago) the *La main à la pâte* (Hands on) programme, intended to revitalize the teaching of sciences in primary school in France (www.lamap.fr/, www.elearningeuropa.info/). In this paper we present an exercice where the students *manipulate* concepts following the first two principles of the method.

1.1. Our public

We teach specification to first-year students of an IUT. All these students have passed the baccalaureate at the end of secondary school, which in France is the entrance exam for university. Every holder of the baccalaureate can enter a French university without selection. In the case of the IUT there is an exception: for computing a scientific baccalaureate is mandatory. It is interesting to note that for five years now we have had students who don’t know the manual algorithm for division (even though they must have learned and used it in primary school). The proliferation of such a ”black box” culture is certainly a challenge for university teachers.

1.2. Our teaching of formal specification

Our teaching [4] is split into two ”modules”: ”spec1” covers the teaching of the B formal method[1, 3], n-ary relational model. ”spec2” covers automata, process algebra, Petri nets, Grafcet, sequence charts, event B. We use the following tools : Atelier B, ProB, LTSA. We use the following exercise for the ”spec1” module, when the students are accustomed to the B notation. In this paper, we just detail some parts of our exercise and name the concepts this exercise allows us to introduce.

1.3. Our goals when we use the abacus

Our goals when we use the abacus is to teach students to

- use basic notions of computing as numerical notation, number representations
- practise observation, abstraction
- use formal notation
- introduce the concept of refinement
- perform reverse engineering

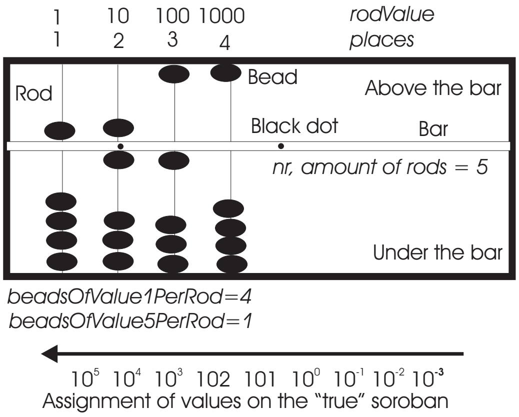

FIGURE 1: *European-style implementation of One hundred sixty five on the soroban*

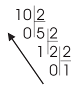

FIGURE 2: Calculus of the binary representation of the number *ten*

1.4. Some types of abacus

Amongst the best known [7], there are: the Russian abacus, (*stochotv*) with 10 beads per rod, the Chinese abacus, (*swan pan*) with a bar separating every rod into two parts, one with 2 beads, the other with 5 beads, the Japanese abacus (Fig. 1), (*soroban*) the object of this study, with a bar separating every rod in two parts, one with 1 beads, the other with 4 beads.

1.5. The problem given to the students

We give the soroban to the students and we ask them how a number is written on the soroban and how an addition is done. We also give them a *stochoty* and a *swan pan*. They can use an overhead projector to show the manipulations done on the *soroban* on a screen. We ask them to observe the soroban methodically, to use the principles and elementary knowledge presented in the first courses in computing, a knowledge which is acquired in elementary school (ten years ago, numerical writing was taught to 10-year-old pupils. They were able to write number in different bases). We gave the same problem to students with four years of studies in computing behind them. There was no noticeable difference between their performance and that of the first-year students.

2. THE DIFFERENT PHASES OF THE EXERCISE

2.1. Abstraction from the soroban

We ask the students to abstract the essence of the soroban graphically - that is, we ask them for a graphical model in the sense of Minsky (*”To an observer B, an object A* is a model of an object A to the extent that B can use A* to answer questions that interest him about A”*) [5]. And we ask them to name the elements of this model: rods (vertical), bar (horizontal), beads. On the bar, we find black dots. In fact they have the role of commas sometimes used in writing numbers in the U.K. In accordance with the international convention, the decimal point should be represented by either a dot or a comma (example: 3.14 or $3,14$), and digits should be grouped in threes without further punctuation on either side. However, in some countries, some people will write 12 545, 15 as $12,545.15$. The black dots on the bar have the role of the blank in this last writing. We abstract (we do not put in our model) these black dots. It is the occasion to make the difference between an interface for the user (with properties as readability) and the core of the machine.

Numbers are written with ”signs”. We ask the student to try to define what a ”sign” is. We then introduce the ”semiotic triangle” [2] to present the difference between signifier, signified and referent. We choose the classical example used during the Middle Ages and at the beginning of the book ”The Name of the Rose” written by Eco: the horse. We also introduce the concept of denotation following Frege with his example: *the morning star, the evening star* denote the same concept (Venus). We apply to numbers. IV, IIII, quatre, four are ”four” ”signifiants” for the same (signifier) concept, the concept of the integer ”four”. We let the student understand that we are not interested by what is denoted by this concept. Following Saussure, we consider that the sign is the pair (signifier/signified) and is arbitrary. But we give to the students the hieroglyphs and their translation to show how the picture of the number was, in the ancient Egypt, related to a meaning. On this occasion, we ask the student to try to define numbers with the concept of cardinality. We introduce them to the Peano axioms and to algebraic specifications. We show that we can have an axiomatic style even with a B specification. For that we use the example of the stack written in two styles in B. Writing a number is like writing other words. So we ask the students to use the concepts relative to language: vocabulary, syntax. A language can be ”a graphical language” (the boxes and arrows of SADT as an example), a ”physical language” with rods, beads etc. In our case, we can consider: Vocabulary[1] = $ba, bu1, bu2, bu3, bu4, r1, r2, r3, r4, on, off$, Words $\{(r1, ba, off), (r1, bu1, on), (r1, bu2, on), (r1, bu3, off), (r1, bu3, off), ...\}$. Following Saussure, we emphasise the difference between *diachrony* (study of the system evolution, of the language evolution) and *synchrony* (the study of the system in its present state). The historic evolution of the abacus is diachrony: the number of beads has changed to arrive at a minimal set of beads. In fact, the students do not follow our advice or hints. They do not arrive at the solution after one hour. We forbid them to use the Internet. One of the goals of this exercise is to learn to be autonomous; to learn that what is taught can be useful. So we are forced to ask them questions to lead them to the solution of the problem. In the next section we present the scenario we give them.

3. SCENARIO OF THE DISCOVER

3.1. What number system ?

The numbers are written following a number system. Do the Japanese use the decimal number system? How can we find out? A solution is to consult the part of a device documentation written in the Japanese language. It is easy to find. Then we note that inside Japanese texts we find numbers evidently written in decimal notation with Arabic numerals (the dates, the number of pieces, etc.). What are the principles of number systems (as the binary, decimal, octal, hexadecimal, etc.)? If they do not give a response, we ask them to perform an arithmetic operation with the Roman-numeral system. If we still do not obtain a response, we lead them through the following points:

1. What is the vocabulary of our decimal system? $V = \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9\}$ What is the base? $card(V) = 10$ [2]
2. To write ”zero”, we use 0, to write ”one”, we use 1, to write ”nine”, we use 9 ...
3. Now we have to write ”ten”. The inspired idea has been to say ”we do not have another element in our vocabulary, but we have plenty of room. The place used until now, had (has always!) the value ”one””.
4. ”zero” = ”zero” times ”one ”, ” one ” = ” one ” times ” one ”, etc. The place near the first place used will have the value for which I have no member of my vocabulary, *i.e.* ”ten”. Now I can continue.
5. To write ”ten”, we write 10 *i.e.* 10 = ”zero” times ”one” + ”one” times ”ten”
6. To write ”eleven”, we write 11 *i.e.* 11 = ”one” times ”one” + ”one” times ”ten”, etc.

Very rarely a student says that the numbers are written as: $(1 \times base^0 + 2 \times base^1 + 3 \times base^2 +)$. They can observe that they have difficulties to use what has been studied in mathematics. When we deal with students, groups, orders, invoices, etc. they say: ”we have never seen that in maths”.

[1] *ba* is for bead above, *bu* is for bead under, *r* for rod
[2] Note: The vocabulary of Maya number system is $\{shell, dot, bar\}$ and the base is *base-twenty*.

But when we deal with number systems, ”we are in maths”! For several students this experience changes their attitude toward specifications. And it is one of our goals with this exercise. Then we name this number system: the place-value system.

3.2. Reading and writing

What place shall we take when we need a new place, the place on the right or the place on the left? You can observe that we took the place on the left. Curious, we write and read from the left to the right! It is the occasion to ask the question ”what is writing?” The students give different definitions. These are tested with counter-examples. To help the students, we ask them how ”reading” and ”writing” are taught in nursery school. The pupils have to put pictures in sequence to tell a story. The proper character of a writing/reading is the fact that words, phrases and texts are sequences. But there are languages where one writes (and reads) right to left, Arabic for example. We know that the place-value system reached the West in the Middle Ages through translations of Arabic mathematical texts (the origin of the place-value system being India). And that’s why we write numbers in the opposite direction of the direction we write the other words and phrases. It is the occasion to ask the student to give the manual algorithm to translate a number from the decimal notation to the binary notation (they learn that in one of their first lessons at the IUT). And then to make them observe that they form the result following the arrow of figure 2.

3.3. Writing numbers on the soroban

How are the elements of the vocabulary written on the soroban ? We know that the invention of the ”zero” was very important. So how to write, ”zero”? We have beads moving on the rods. The abacus shows a bar parallel to its largest sides. This bar separates the abacus into two parts: for the above part, the rods have 4 beads, for the others (under the bar), the rods have only one bead. We can imagine two solutions a) To push all the beads into contact with the bar b) To push all the beads into contact with the frame. We will experiment to know if we can find the right answer. First, we will choose a). Do all the beads of the same rod have the same value? If we answer ”yes” and if we give an ”effective” value to a bead when we make this bead touch the bar (A bead can touch the bar either directly or by being in contact with a bead ”touching” the bar), in this way we can write $\{0, 1, 2, 3, 4, 5\}$. We can also write $\{0, 9, 8, 7, 6, 5\}$ or $\{0, 9, 2, 8, 7\}$...But, let us formulate a hypothesis: the inventors of the abacus have made it simple, applying the Occam razor. It seems that we are in base 5 and not in base 10. With the lower part of the abacus we can write the numbers $0..4$. How to do that? We can think that the bar is used to ”put on” the numbers (our beads). The first bead of the rod touching the bar without another bead touching this bead represents ”one”. ”Two” will be represented by two beads in contact with one of them in direct contact with the bar, etc. But we still have to write ”five”, ”six”, etc. We have no more beads under the bar, but we have one bead above the bar. We will now need a ”five”. We give value ”five” to this bead. To write ”five” we move a bead into contact with the bar, and move the bead under the bar in contact with the frame. Now come back to our hypothesis b). If we apply it, ”zero” is written when no bead touches the bar. All the beads touch the frame. First we point out that there is no state for a bead where it does not touch the frame and does not touch the bar then we ask the students to specify the property formally. We do not obtain a more complicated procedure. We ask the students what we can mean by ”complicated” ? We agree on ”with no more manipulations than the first one”. So, what is the right answer? If we observe the bar, we see the black dots...and we can imagine the role indicated at the beginning of this presentation. Our scenario continues by covering addition and subtraction of numbers. In parallel we specify using the B notation. We then introduce the refinement and the proof of refinement. For instance, we use the *swan pan* to give examples of implementations (see figure 3). The two following tables (the first for the *soroban*, the second for the *swan pan*) illustrate answers to questions [6] we ask students. We ask the students which positional systems are better than others. Then we suggest the following answer: base of primes to have more uniform algorithms; base with many divisors for business and social activities (e.g. 12); 2 for computers; 8 for men and computers.

Value of the top beads	Value of the bottom beads	Max nb we can write	Writing not unique
5	1	9	-
1	2	9	-

Value of the top beads	Value of the bottom beads	Max nb we can write	Writing not unique
6	1	17	-
5	1	15	5 and 10

4. SPECIFICATION IN B

We specify the machine SOROBAN. The parameter nr of the machine is the amount of rods. The state of the abacus is represented by the function $myNumberImp$. The element $(1 \mapsto (1 \mapsto 0))$ of this set means that on rod 1, there is one bead of value 5 on the bar and zero beads of value 1 on the bar. Note that our numbering of the rods is done like in Europe, from left to right. But it is easy to "reverse" an abacus! It is the occasion to analyse the direction of the reading in different natural languages and to show that we have to pay attention to the discourse in natural language. Ancient Egyptians used (not always) a head hieroglyph oriented in the direction of the reading and marking the beginning of the text and a "tail" hieroglyph marking the end of the text. For the Japanese, it is said that *the reading is top to bottom and right to left*. (But in scientific books, it is the same as in Europe). If we turn the page $90°$ to the left we have exactly the same configuration as in Europe. It is very different from the Arabic language. In the Arabic language, the writing is right to left. And the reading is right to left. When the reading/writing sequence in English is $[S, O, R, O, B, A, N]$, it is also $[S, O, R, O, B, A, N$ in Japanese but it is $[N, A, B, O, R, O, S]$ in Arabic language. The machine NUMBER can be considered as being an abstract machine that can be refined. It can alternatively be considered as being abstracted from the more concrete machine SOROBAN. The machine A4RODSSOROBAN gives the translation from binary to decimal notation.

MACHINE(nr) SOROBAN
CONSTRAINTS $nr \in 14..50$
CONSTANTS $rodValue, beadsOfValue1PerRod, beadsOfValue5PerRod, places$
PROPERTIES
$places \in 1..nr \ \wedge rodValue = \{(places \mapsto value) | places \in 1..nt \wedge value = 10^{places-1}\}$
$\wedge beadsOfValue1PerRod = 4 \wedge beadsOfValue5PerRod = 1$
VARIABLES $myNumbImp$ /* my number implemented on the soroban */
INVARIANT
$myNumbImp \in 1..nr \rightarrow \{0, value5\_beadPerRod\} \times 0..value5\_beadPerRod$
OPERATIONS
$mb \leftarrow hereIsMyNumbSoroban \mathrel{\widehat{=}} mb := myNumbImp;$
$mn \leftarrow plusse \mathrel{\widehat{=}} \ldots$
$mn \leftarrow moinsse \mathrel{\widehat{=}} \ldots$
END

MACHINE(max) NUMBER
CONSTRAINTS $max \in \mathbb{N} \wedge max < 10^{51} - 1$
VARIABLES $myNumber$
INVARIANT $myNumber \in 0..max$
OPERATIONS
$mn \leftarrow hereIsMyNumber \mathrel{\widehat{=}} mn := myNumber;$
$mn \leftarrow plusse \mathrel{\widehat{=}}$ PRE $myNumber < max$ THEN $myNumber := myNumber + 1$
$\| \ mn := myNumber + 1$END;
$mn \leftarrow moinsse \mathrel{\widehat{=}}$
PRE $myNumber > 0$ THEN $myNumber := myNumber - 1 \ \| \ mn := myNumber - 1$END END

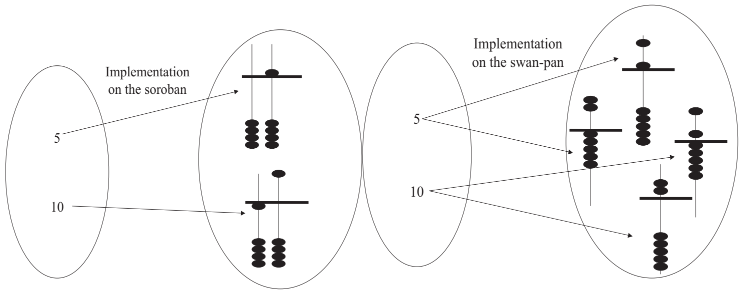

FIGURE 3: Implementation on the *swan-pan* (right) and on the *soroban* (left)

MACHINE A4RODSSOROBAN
CONSTANTS $rodValue, numberImp$
PROPERTIES
$numberImp \in 1..4 \rightarrow \{0,1\} \times 0..4 \wedge rodValue \in \{1 \mapsto 1, 2 \mapsto 10, 3 \mapsto 100, 4 \mapsto 1000\} \wedge$
DEFINITIONS
$inter \mathrel{\widehat{=}} \{kk | \exists (xx, yy, zz) \bullet ((xx \mapsto (yy \mapsto zz)) \in myNumberImp \wedge (kk = ((yy + zz)$
$\times rodvalue(xx))))\}$
VARIABLES $inter, dec, myNumbImp$
INVARIANT
$myNumbImp \in NAT \wedge dec = \sum kk \bullet (kk \in inter | kk)$
INITIALISATION
$myNumbImp := \{1 \mapsto (0 \mapsto 2), 2 \mapsto (1 \mapsto 1), 3 \mapsto (1 \mapsto 2), 4 \mapsto (0 \mapsto 3)\} \parallel dec := 3762$
OPERATIONS
$writeNumber(mn) \mathrel{\widehat{=}}$
PRE $mn \in 1..4 \rightarrow \{0,1\} \times 0..4$ THEN $myNumbImp := mn$ END;
$rep \leftarrow value \mathrel{\widehat{=}} rep := dec$
END

5. CONCLUSION

In this paper we showed how the teaching method ”La main à la pâte” could be used to teach formal specification. Observation, abstraction, manipulation are considered as complementary to calculus and formal verification.

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